

Introduction to Problem Solving

The following materials was taken from several sources.
It was not created by ELMACON organizers

Four Principles

How to Solve It suggests the following steps when solving a Mathematical Problem:

- Understand the problem
- Make a plan
- Carry out the plan
- Look back on your work. How could it be better?

Second principle: Devise a plan

- 1. Use Diagrams / Models
- 2. Act it Out
- 3. Use Before & After
- 4. Use Systematic Listing
- 5. Look for Patterns
- 6. Work Backwards
- 7. Use Guess & Check
- 8. Simplify the Problem
- 9. Make Supposition
- 10. Solve Part of the Problem
- 11. Paraphrase the Problem

Danny raises some chickens and rabbits in this little farm. These animals have 15 heads and 40 feet altogether. How many chickens and rabbits does he raise?

Step One: Understand

- How many chickens and rabbits are there altogether?
- How many feet are there altogether?
- How many feet does each chicken have?
- How many feet does each rabbit have?

Problem Solving

Problem → Diagram → Calculation

- *Mathematical concepts or problems sometimes require an illustrative calculation.*
- *Mathematical Concepts can occur in a variety of situations, giving the opportunity to use what is known.*

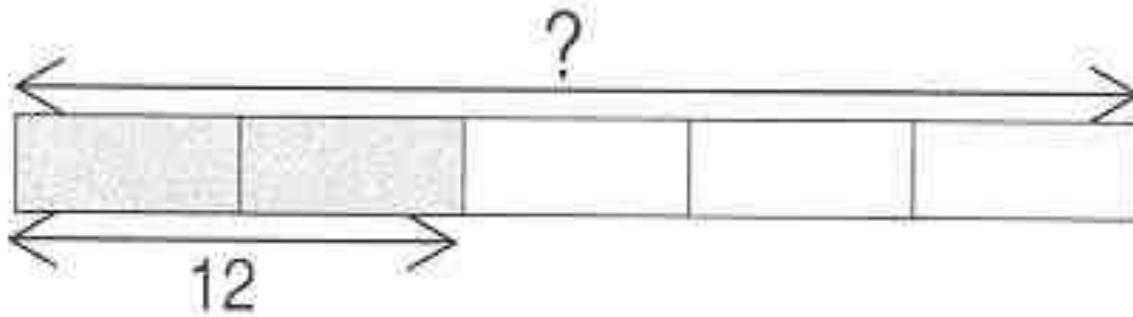
Kathy bought a blouse for three times as much as her scarf cost. The scarf cost half as much as her hat. Her hat cost \$10.00. How much did her blouse cost?

Use of these models to solve problems

$\frac{2}{5}$ of a number equals 12. What is the number?

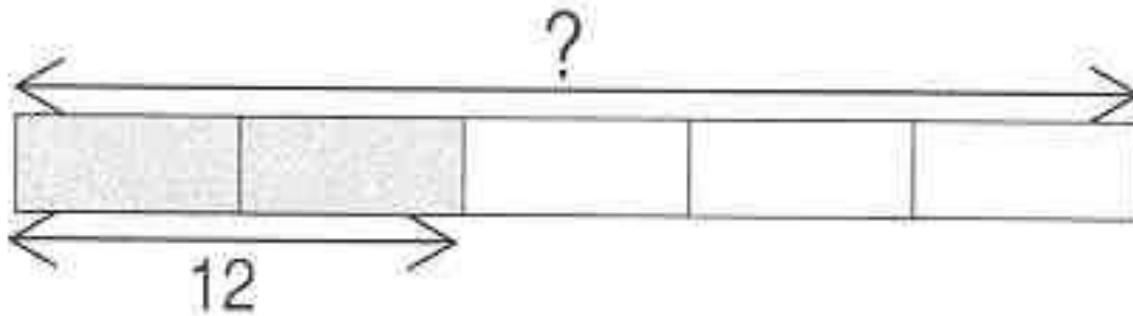
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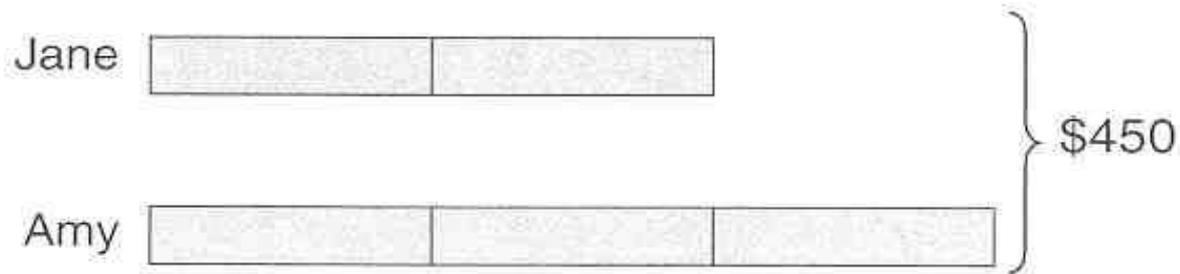
2 units \longrightarrow 12

1 unit \longrightarrow $12 \div 2 = 6$

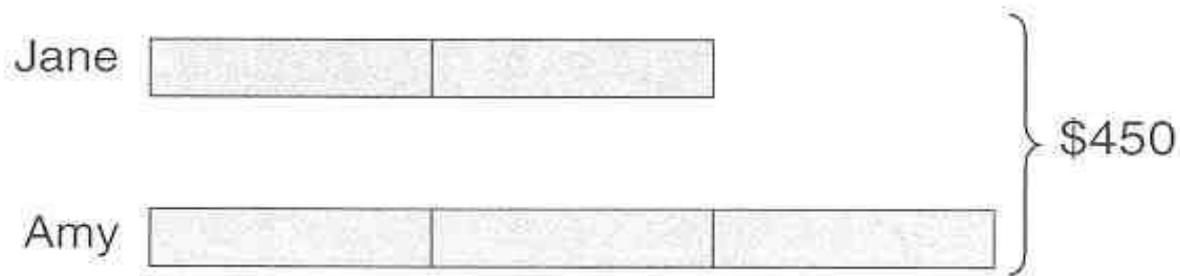
5 units \longrightarrow $5 \times 6 = 30$

Jane's savings are two thirds of Amy's savings. Together they save 450 dollars. How much money did Jane save?

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$$5 \text{ units} \longrightarrow \$450$$

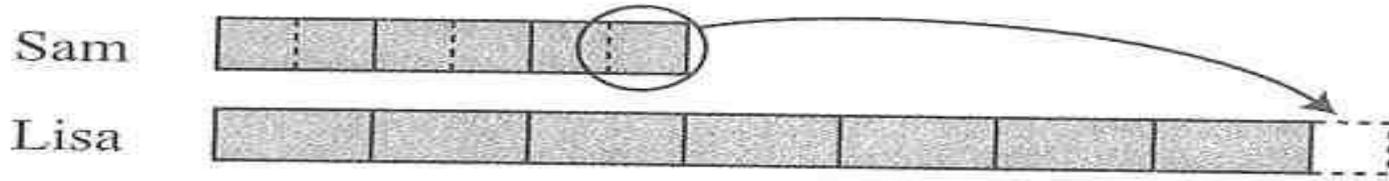
$$1 \text{ unit} \longrightarrow \$450 \div 5 = \$90$$

$$2 \text{ units} \longrightarrow 2 \times \$90 = \$180$$

Sam has $\frac{3}{7}$ the amount of marbles that Lisa has. Sam gives Lisa $\frac{1}{6}$ of his marbles. What will be the new ratio between the number of marbles of Sam and Lisa?

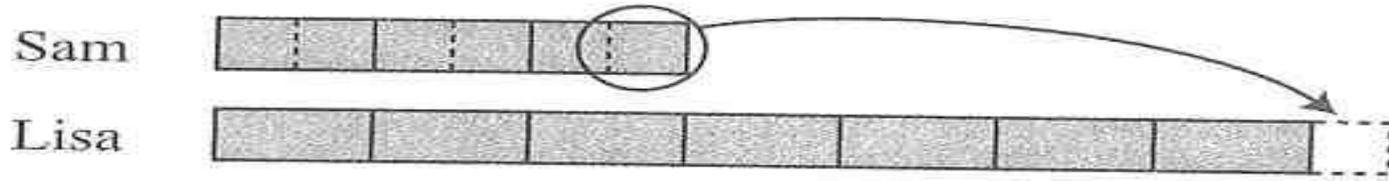
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Before:

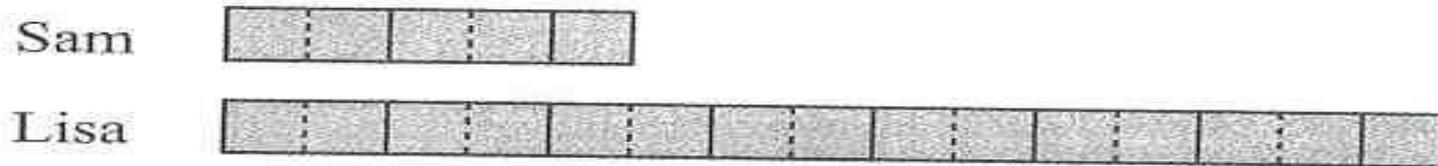


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Before:



After

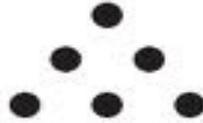




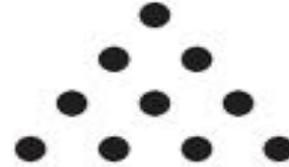
1



3



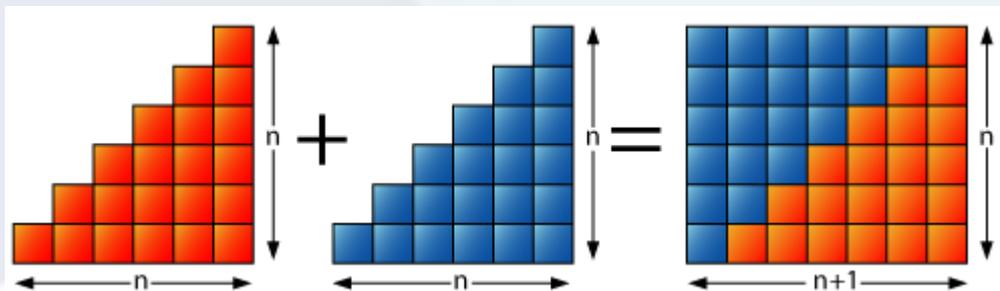
6



10

The first four triangular numbers are 1, 3, 6 and 10.

What is the 10th triangular number?



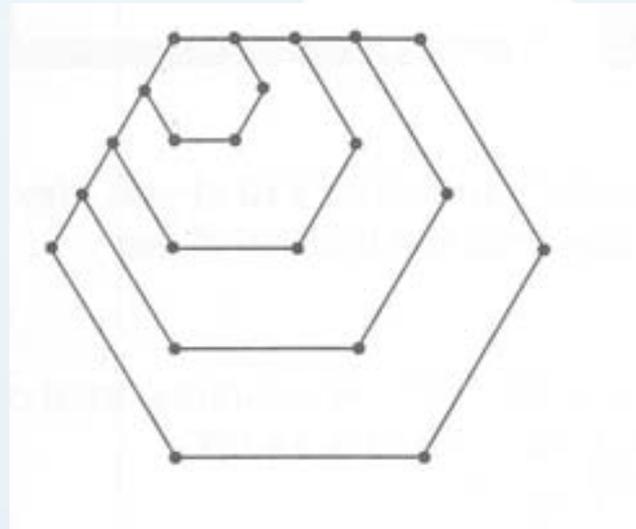
• *Let's find the value of :*

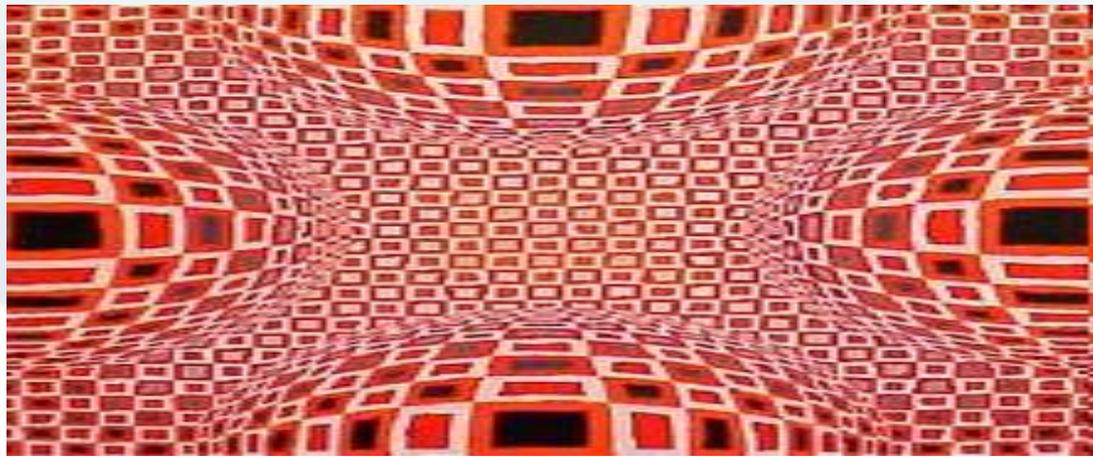
$$1 - 2 + 3 - 4 + 5 - 6 + \dots 99 - 100$$

• *Let's find the value of :*

$$1 + 2 + 3 + 4 + 5 + 6 + \dots 99 + 100$$

Robert is bored and started drawing one hexagon, then kept drawing larger and larger hexagons. How many dots he would have altogether after the 8th hexagon?





A piece of wire 52cm long is cut into two parts.

Each part is then bent to form a square. The total area of the two squares is 97 cm².

How much longer is a side of the larger square than a side of the smaller square ? (Consider only integers for the lengths of the sides.)

Solve for the variable. .

$$2(x + 5) - 7 = 3(x - 2).$$

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$$2x + 10 - 7 = 3x - 6$$

$$2x + 3 = 3x - 6$$

$$2x + 3 - 3x = 3x - 6 - 3x$$

$$-x + 3 = -6$$

$$-x + 3 - 3 = -6 - 3$$

$$-x = -9$$

$$\frac{-x}{-1} = \frac{-9}{-1}$$

$$x = 9$$

It takes 1.5 hours for Tim to mow the lawn. Linda can mow the same lawn in 2 hours. How long will it take John and Linda, work together, to mow the lawn?

Let t be the time for John and Linda to mow the Lawn.

The work done by John alone is $t \times (1 / 1.5)$

The work done by Linda alone is given by $t \times (1 / 2)$

When the two work together, their work will be added. Hence

$$t \times (1 / 1.5) + t \times (1 / 2) = 1$$

Multiply all terms by 6

$$6 (t \times (1 / 1.5) + t \times (1 / 2)) = 6$$

Simplify $4 t + 3 t = 6$

Solve for t

$$t = 6 / 7 \text{ hours} = 51.5 \text{ min.}$$

Percent means per 100, or divided by 100. Dividing by 100 moves the decimal point two places to the left.

$$24\% = \frac{24}{100} = .24$$

To convert a fraction or decimal to a percentage, multiply by 100:

Multiply the fraction by 100 to give the result as a percentage value.

$$\frac{1}{5} \times 100 = 5 \frac{20}{100}$$

To convert a percent to a fraction, divide by 100 and reduce the fraction (if possible):

Divide the percentage value by 100 and simplify the fraction if necessary.

$$60\% = \frac{60}{100} = \frac{3}{5}$$

12 people out of a total of 25 were female. What percentage were female?

Multiply by 100. Dividing the top and bottom by 25 (cancelling) leaves 12×4 .

$$\frac{12}{25} \times \frac{100}{1} = 48\%$$

The price of a \$1.50 candy bar is increased by 20%. What was the new price?

Multiply the price by 20% (20/100). Add the result to the original price. ($\$1.50 + .30 = \1.80)

$$\$1.50 \times \frac{20}{100} = \$0.30$$

$$\$1.50 + \$0.30 = \$1.80$$

A brand new movie just came to Golden Theater. The theater has a goal of selling 2000 tickets for this movie on the 1st week.

The first 2 days it sold 30% of its goal tickets. What is the average number of tickets the theater needs to sell in the next 5 days in order to meet their goal?

The next 5 days sales needs to be $2000 \times (100\% - 30\%) = 1400$ tickets;

Average daily sales = $1400 / 5 = 280$ tickets

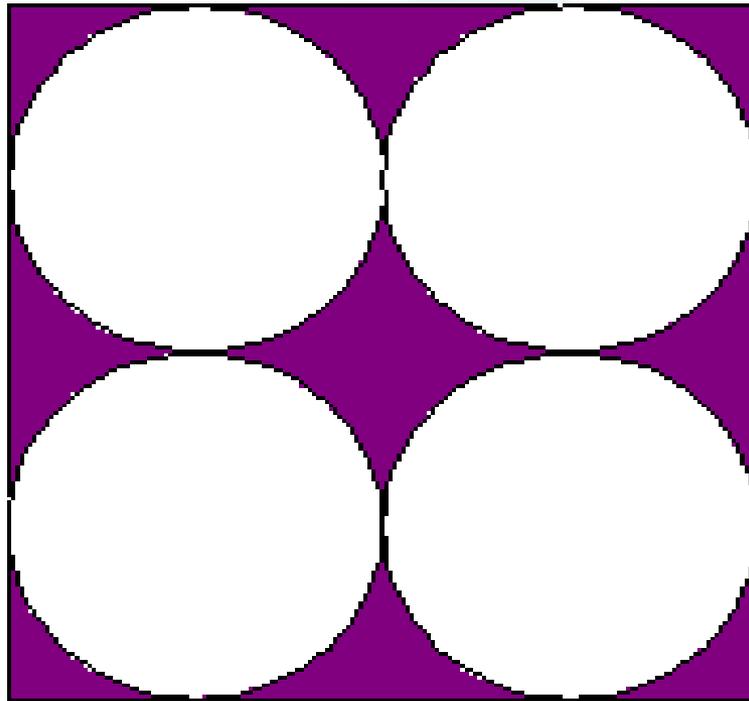
October 1st of 2025 will be on Wednesday.
What day of the week is Christmas day,
December 25th, 2025?

Solution:

There are $31 + 30 + 25 - 1 = 85$ days from
October 1st to December 25th. There are 7
days in a week. $85 \div 7 = 12$ remainder 1;
Therefore December 25th is on a Thursday.

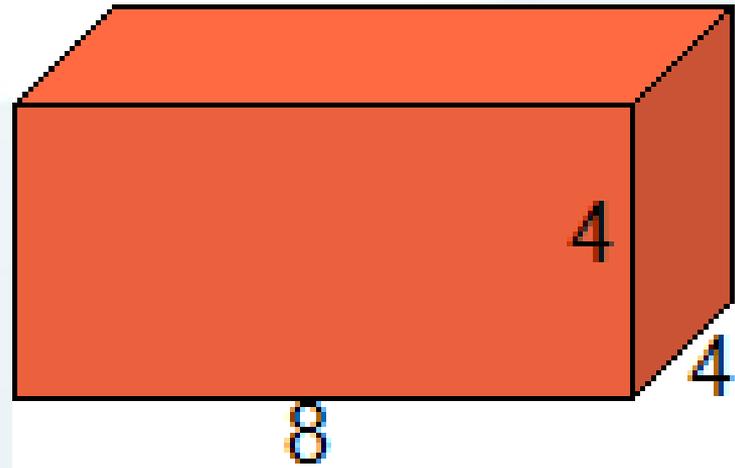
The diameter of the circles is 4 centimeters.
Calculate the area in purple.

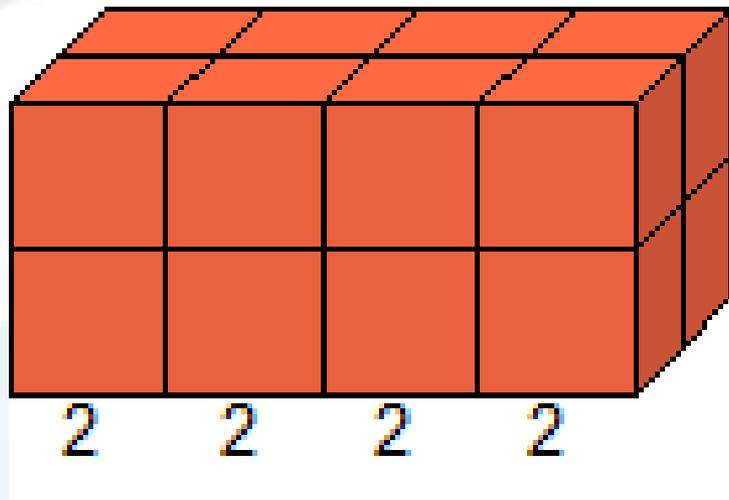
The area is $8 \times 8 - 4 \times (3.14 \times 2 \times 2)$ square
cm.



A rectangular prism is 8 centimeters long, 4 centimeters wide and 4 centimeters tall.

- a) How many $2 \times 2 \times 2$ cm cubes can the prism be cut into?
- b) What is the total surface area of all the $2 \times 2 \times 2$ cm cubes?
- c) c) What is the total volume of all the $2 \times 2 \times 2$ cm cubes?





a) 16

b) 1 cube has surface area of $(2 \times 2) \times 6 = 24$ square cm;

16 cubes have surface area of $24 \times 16 = 384$ square cm

c) $4 \times 4 \times 8 = 128$ cubic cm, (same as the volume of the original prism)

- A train is 300 meters long. A tunnel is 2900 meters long.
- The train travels at 600 meters per minute.
- How long is the time between the moment the front of the train enters the tunnel and the moment the end of the train exits the tunnel?

The length the train needs to travel is $2900 + 300 = 3200$ meters,
Therefore, it takes $3200 \div 600 = 8$ minutes for the train to pass the tunnel.

The following 3 problems are from the
*Mathematical Olympiads for Elementary
and Middle Schools, Inc*

Reduce the complex fraction to a common fraction in lowest terms:

[Hint: Start with $1 - \frac{1}{5}$]

$$\frac{2}{3 + \frac{4}{1 - \frac{1}{5}}}$$

A palindrome reads the same forwards and backwards.

The number 2017102 is a 7-digit palindrome.

Let A represent the least palindrome greater than 2017102.

Let B represent the greatest palindrome less than 2017102. Find $A - B$.

Changing the digit in the units column forces the digit in the millions column to change.

Likewise, changing the digits in the tens or hundreds columns changes the digits in the hundred-thousands or ten-thousands columns.

Each of these changes would bring the result further from the original number than changing the number in the thousands column.

Thus the two closest palindromes to 2017102 would be 2018102 and 2016102. These numbers are **2000** apart.

Alexi opens his favorite mathematics puzzle book and notes that the product of the page numbers facing him is 132,.

Find the sum of these two page numbers.

[Note: All pages are numbered consecutively.]

Strategy: Use the idea that the square root will be close to the answer .

The two page numbers will be consecutive with one slightly less than the square root and the other slightly larger.

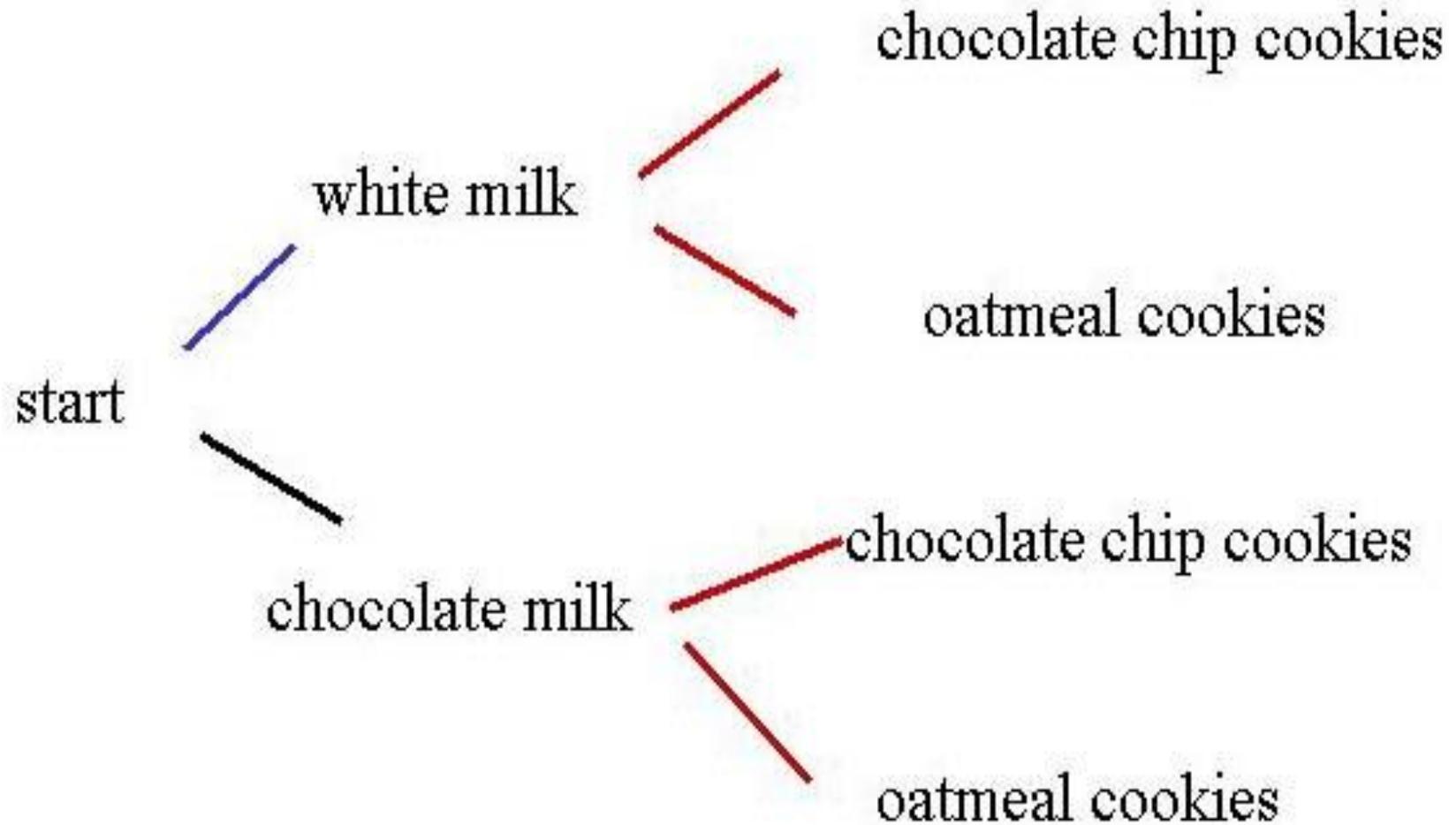
Using any convenient method, determine that the square root of 132 is slightly larger than 11. The two page numbers are 11 and 12.

The sum of these numbers is $11 + 12 = \mathbf{23}$.

Tree Diagram

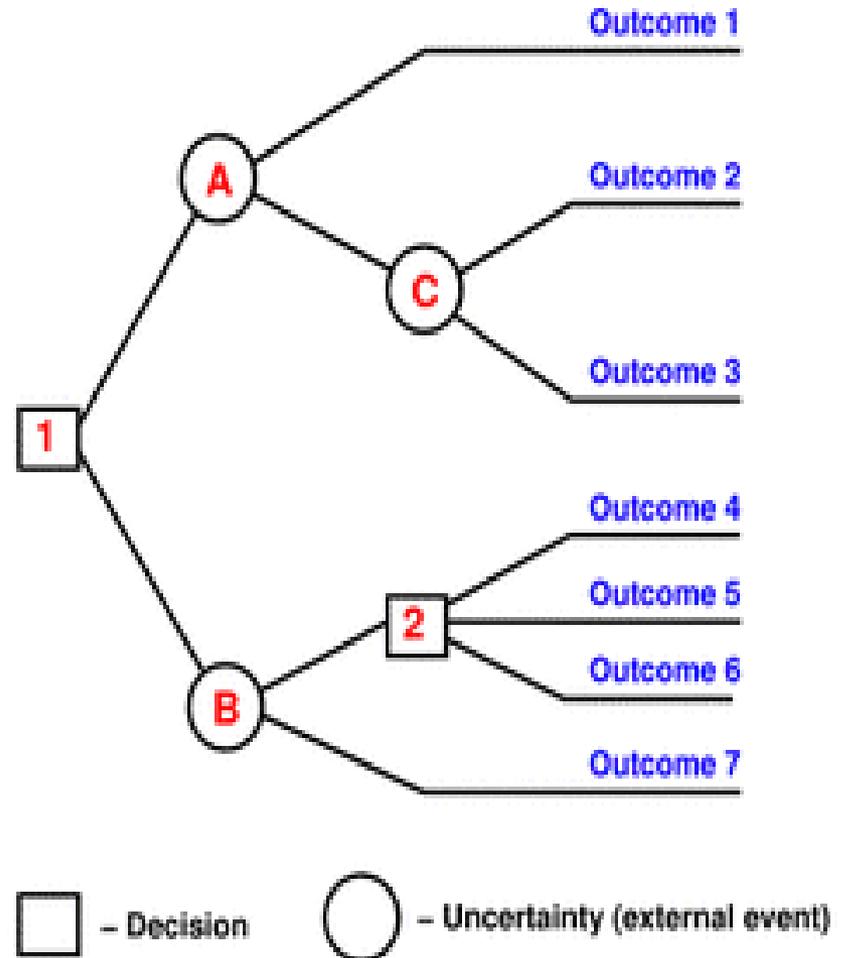
A tree diagram is simply a way of representing a sequence of events. Tree diagrams are particularly useful in probability since they record all possible outcomes in a clear and uncomplicated manner.

Tree Diagram?



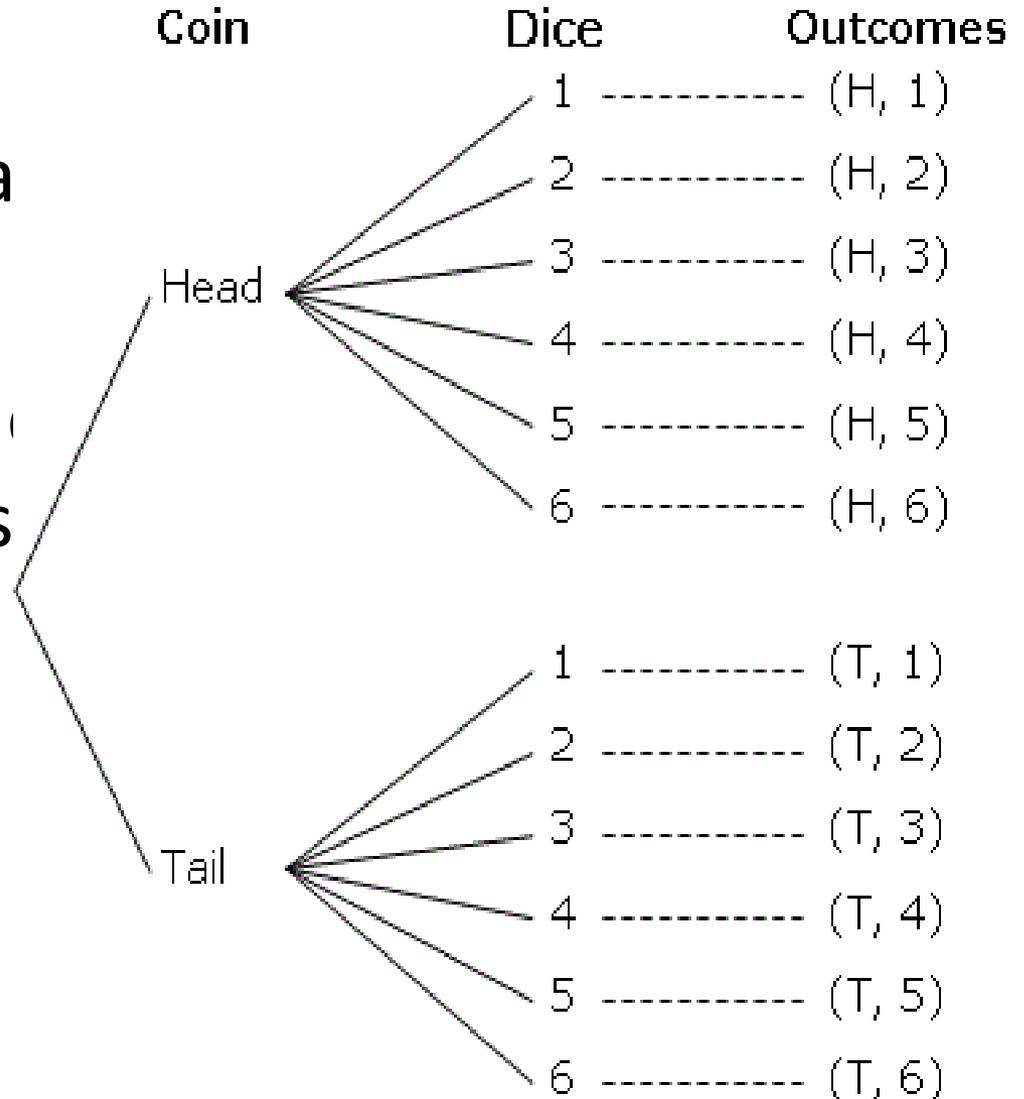
Decision Tree

- Decision Trees help you choose between multiple outcomes/courses you might take. They are very visual and help the user understand the risks and rewards associated with each choice.



Probability problems

- A coin and a dice a
- We can use a tree (possible outcomes)



Trigonometry

degrees	radians	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\csc \theta$	$\sec \theta$	$\cot \theta$
0°	0	0	1	0	-	1	-
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$
90°	$\frac{\pi}{2}$	1	0	-	1	-	0

- Sine: $\sin(\theta) = \text{Opposite} / \text{Hypotenuse}$
- Cosine: $\cos(\theta) = \text{Adjacent} / \text{Hypotenuse}$
- Tangent: $\tan(\theta) = \text{Opposite} / \text{Adjacent}$

