First ELMACON Review
basic concepts

Prime Factorization

- A Prime Number can be divided evenly **only** by 1 or itself. And it must be a whole number greater than 1.
- •The first few prime numbers are: 2, 3, 5, 7, 11, 13, and 17
- •"Factors" are the numbers you multiply together to get another number:

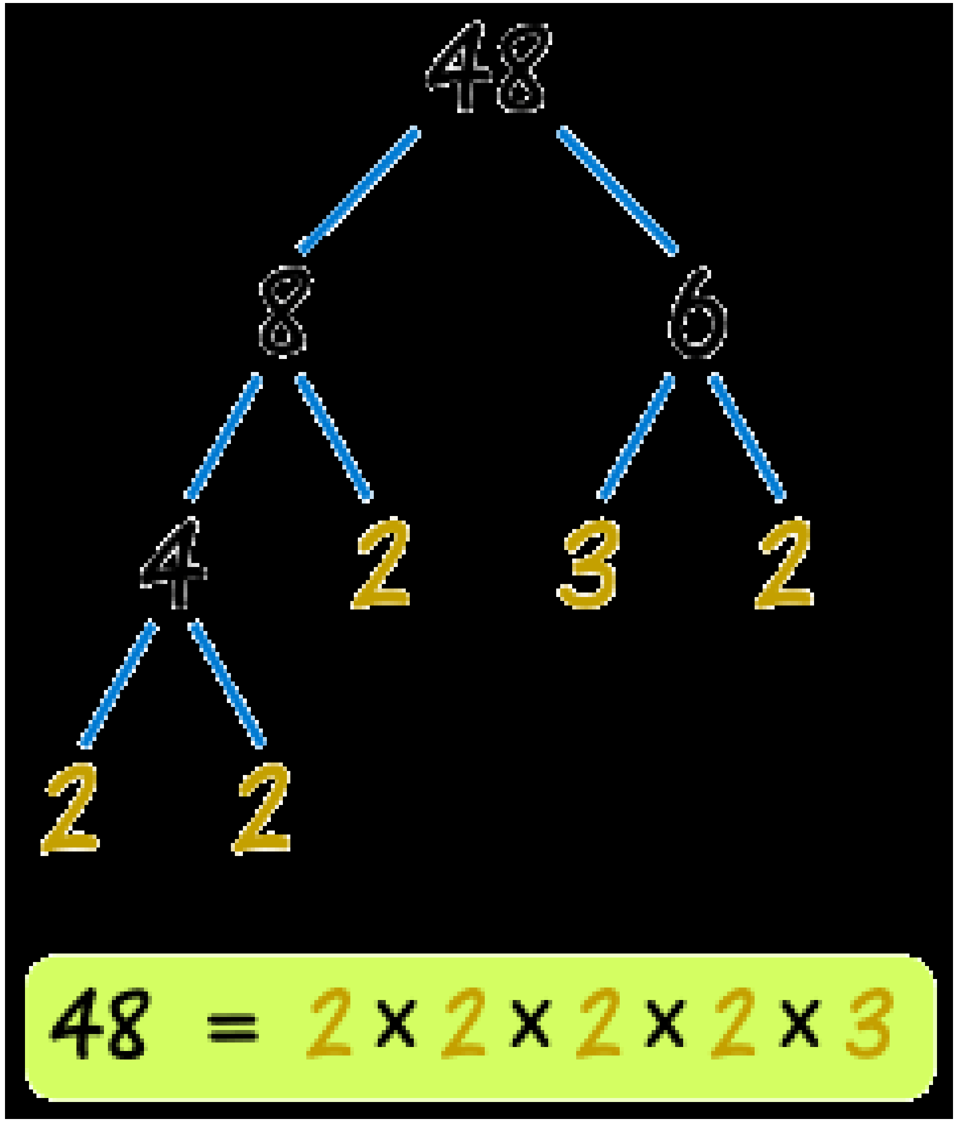
- "Prime Factorization" is finding which prime numbers multiply together to make the original number.
- **What are the prime factors of 12 ?**
- It is best to start working from the smallest prime number, which is 2, so let's check:
- 12 ÷2 = 6
- Yes, it divided evenly by 2. We have taken the first step!

- But 6 is not a prime number, so we need to go further. Let's try 2 again:

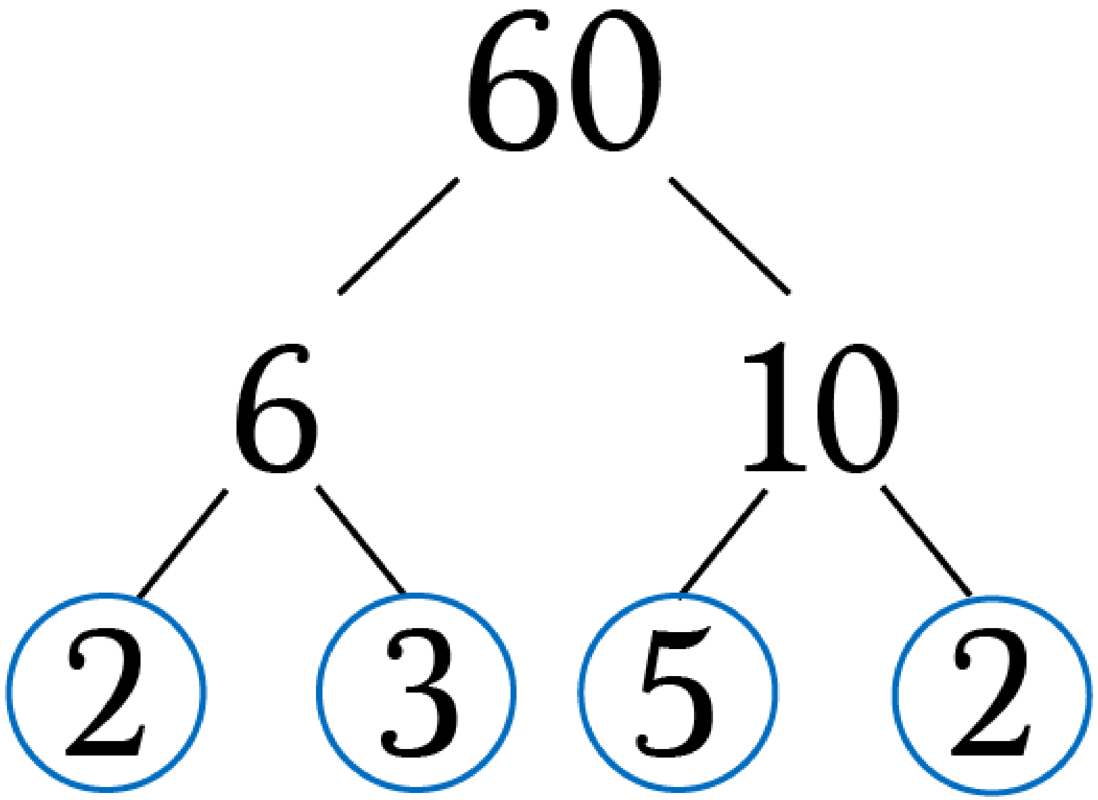
- 6 ÷2 = 3
- •Yes, that worked also. And 3 **is** a prime number, so we have the answer:

- **12 = 2 ×2 ×3**

- "Factor Tree" can help: find any prime of the number, then the factors of those numbers, etc, until we can't factor any more
- **Example: 48**
- **48 = 8 ×6**, so we write down "8" and "6" below 48
- Now we continue and factor 8 into **4 ×2**
- Then 4 into **2 ×2**
- And lastly 6 into **3 ×2**
- We can't factor any more, so we have found the prime factors.
- Which reveals that **48 = 2 ×2 ×2 ×2 ×3**
- (or **48 = 24×3**using exponents)

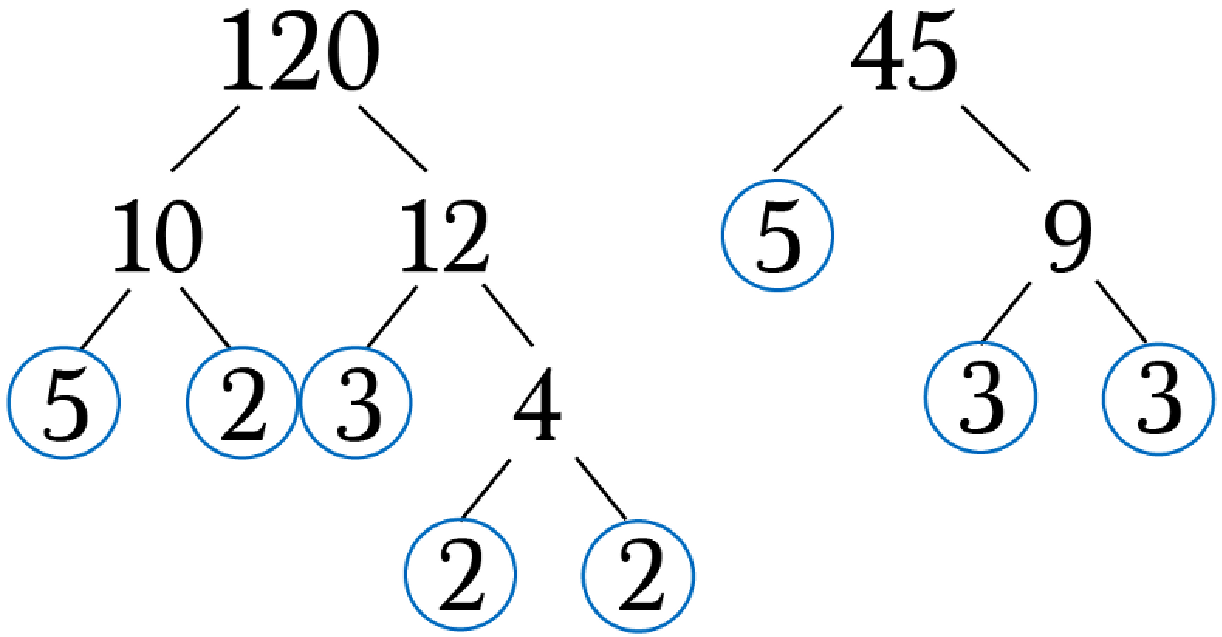


Here is the prime factorization of 60.
Can this help us to find out how many factors 60 has?



Finding the Greatest CommonFactor

- The **GCF** is the largest number that divides into both values without a remainder. Let’s find the **GCF of 120 and 45.**



Finding the Greatest CommonFactor

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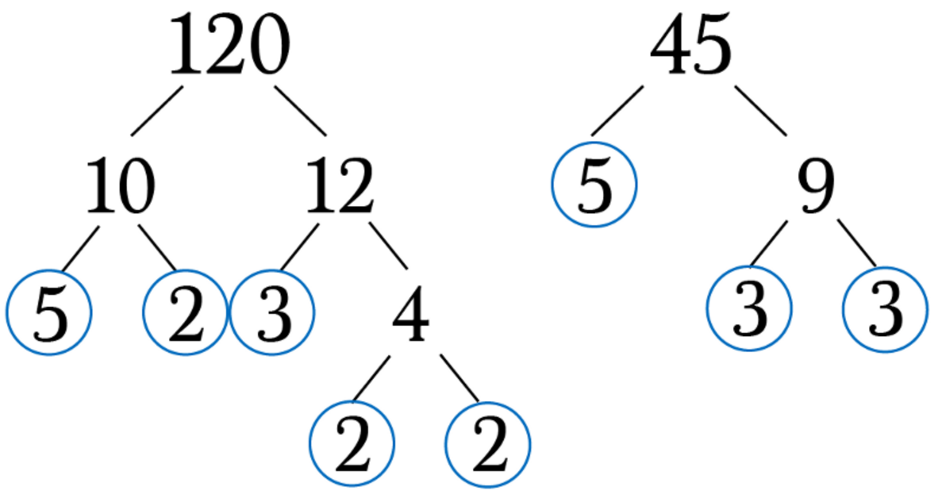


- Factors of 120: **1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60 and 120.**
- Factors of 45: **1, 3, 5, 9, 15, and 45.**
- Greatest common factor is 15.
- Another way: You can find the greatest common factor (GCF) by listing the factors of each number and finding the largest one they have in common

The LCM, least common multiple, is smallest value that two or more numbers multiply into.
Let’s find the LCM of 120 and 45.

- **Multiples of 120: 120, 240, 360, 480, 600, 720, 840……**

- **Multiples of 45: 45, 90, 135, 180, 225, 270, 315, 360, 405, 450**……

- The LCM (45 and 120) is 360

- Is there an easier way to find it?

The LCM, least common multiple, is smallest value that two or more numbers multiply into. Let's find the LCM of 120 and 45.

- Begin by using factor trees to write out each number's prime factorization. We have already found the prime factorizations for 120 and 45:

$$2^3 \cdot 3 \cdot 5 \quad \text{and} \quad 3^2 \cdot 5$$

- The **LCM will be the product of the largest multiple of each prime that appears on at least one list.**For example we have a 2, 3 and 5, so I'll choose the largest multiples of each and find their product.

$$2^3 \cdot 3^2 \cdot 5 = 8 \cdot 9 \cdot 5 = 360$$

- Therefore the least common multiple of 120 and 45 is 360.

- At a summer camp, chocolate milk is served every other day, corn is served every 4 days and pizza every 7 days. Today all three were served. What is the smallest number of days until all three are served again?

- The organizers of a gymnastics event wish to arrange the participants in neat rows. They try rows of 2, 3, 4, 5, 6, 7 and 8, but in each case there is one gymnast left over. There are fewer than 1000 gymnasts in all. How many are there? Explain your reasoning. (Hint: What if 1 gymnast left the room?)

The fundamental counting principle

- If there is a sequence of Independent events that can occur:
  - *a_1, a_2, a_3,...... a_n ways ,*
  - Then the number that all events occur is
  - $a_1 \times a_2 \times a_3 \times \ldots\ldots a_n$

  - How many ways students can answer 3 questions true, false or I don't know.

  - The counting principle tells us, that since we can answer three ways every time
  - The number of ways students can answer is = 3 x 3 x 3 = 27

How many passwords are possible by using 6 digits where the first 2 digits must be letters and the last four digits must be numbers?

- 26 X 26 X 10 X 10 X10 X10 = 6,760,000

A restaurant offers a special menu where people can choose one of each different category.
People can choose: one of 4 beverages, one of 5 appetizers, one of 6 main dishes and one of 5 desserts.
How many different meals are possible?

A gate has a key pad with digits 0 to 9. How many possible code combinations are there if the code is 4 digits long?

- A) If repetition of numbers is allow?

10 x 10 x 10 x 10 = 10,000

- B) If repetition is not allow?

10 X 9 X 8 X 7 = 5040

Let’s talk about factorials

- n! = 1 x 2 x 3 x …… x n

- 5! = 1 x 2 x 3 x 4 x 5

- 10! = 1 x 2 x 3 x 4 x 5 x 6 x 7 x 8 x 9 x 10

- Problem 11

Let's talk about permutations

Permutation is a mathematical calculation of the number of ways a particular set can be arranged, where order of the arrangement matters.

$_n\mathbf{P}_r$ **=** n X (n-1) x (n-2) X ……. (n – r + 1)

$_n\mathbf{P_r} = \frac{\boldsymbol{n}!}{(\boldsymbol{n}-\boldsymbol{r})!}$

How many ways 10 athletes can be awarded 1st , 2nd and 3rd place?

$_{10}\mathbf{P}_3 = \frac{\mathbf{10}!}{(\mathbf{10}-\mathbf{3})!} = \frac{\mathbf{10}\ x\ \mathbf{9}\ x\ \mathbf{8}\ x\ \mathbf{7}\ x\ \mathbf{6}\ x\ \mathbf{5}\ x\ \mathbf{4}\ x\ \mathbf{3}\ x\ \mathbf{2}\ x\ \mathbf{1}}{\mathbf{7}\ x\ \mathbf{6}\ x\ \mathbf{5}\ x\ \mathbf{4}\ x\ \mathbf{3}\ x\ \mathbf{2}\ x\ \mathbf{1}}$ = ***10 x 9 x 8***

Combinations:

- We say that there are ***n*Cr** **combinations** of size **r** that may be selected from among n choices without replacement where order doesn't matter.

$$_{n}C_{r} = \frac{_{n}P_{r}}{_{r}P_{r}} = \frac{n!}{(n-r)!r!}$$

In a lottery a player picks 4 number from 0 to 9 (without repetition) . How many different choices does the player Have?

a) Order matter:

$$_{10}P_{4} = \frac{10!}{(10-4)!} = 5040$$

b) Order does not matter

$$_{10}C_{4} = \frac{10!}{(10-4)!4!} = 210$$

Let's assume that we have 10 balls, and let us say that balls 1, 2, 3 are chosen.
These are the possibilities

- Order does matter:

- 123
- 132
- 213
- 231
- 312
- 321

The permutations have 6 times as many possibilities.

- Order does not matter:

- 123

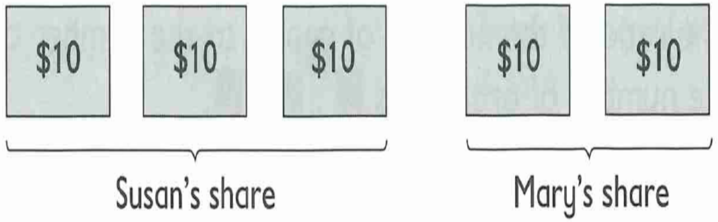
Combination problems:

- How many ways 5 students can be chosen from 12 students?

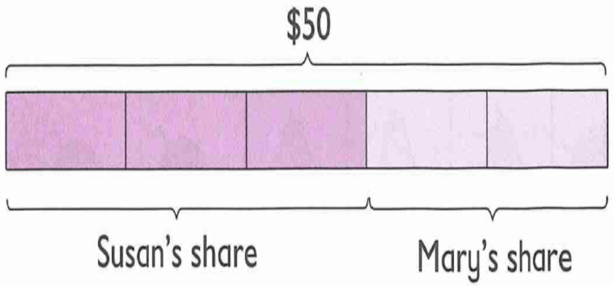
Ratio, Proportion, Percentages and Decimals

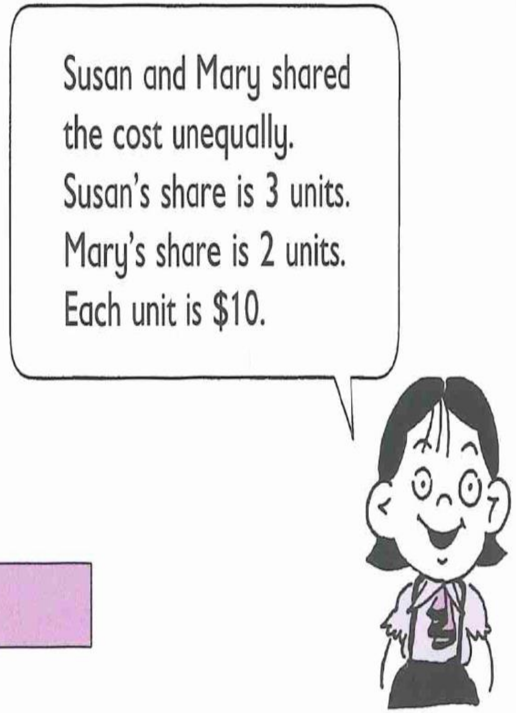
Ratio and Fraction

Susan and Mary bought a present which cost $50. Susan, being the elder sister, paid a bigger share of the cost.

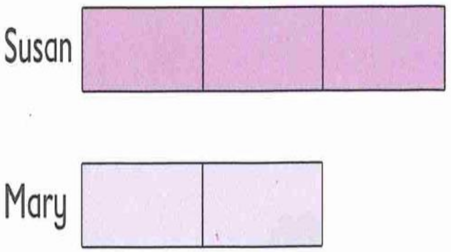


We can also show how Susan and Mary shared the cost like this:



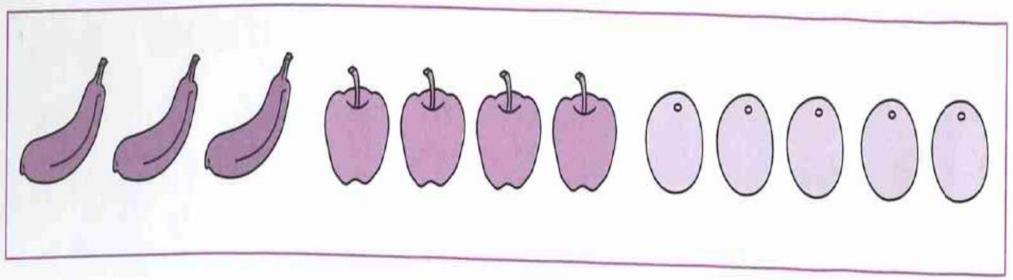


or like this:



The ratio of Susan's share to Mary's share is 3 : 2.

The ratio of Mary's share to Susan's share is ■ : ■.

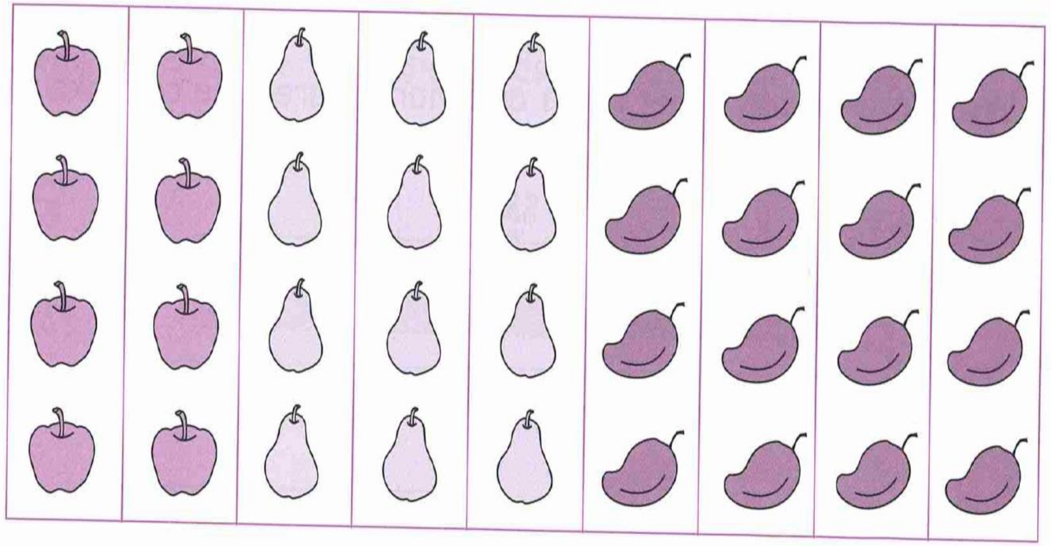


(a) The ratio of the number of bananas to the number of apples is 3 : 4.

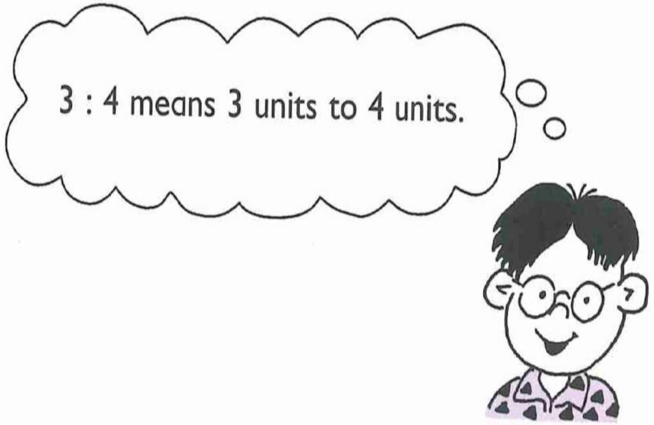
(b) The ratio of the number of oranges to the number of bananas is ■ : ■.

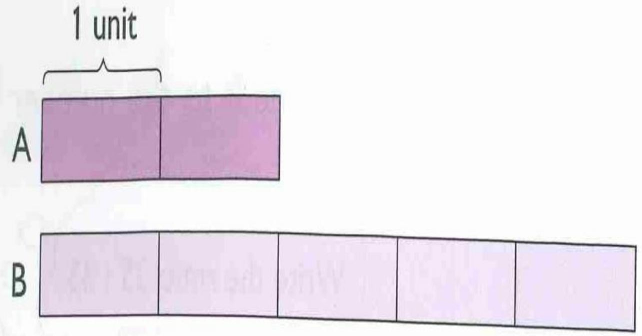
(c) The ratio of the number of bananas to the number of apples to the number of oranges is 3 : 4 : 5.

(d) The ratio of the number of apples to the number of bananas to the number of oranges is ■ : ■ : ■.



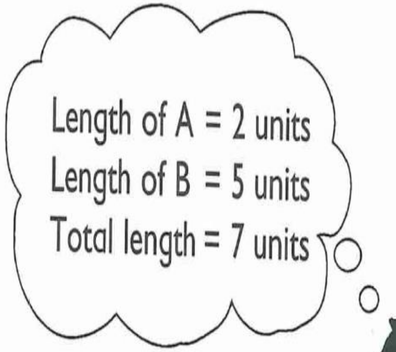
(a) The ratio of the number of pears to the number of mangoes is 3 : 4.





(a) The ratio of the length of A to the length of B is 2 : 5.

(b) The ratio of the length of A to the total length of A and B is 2 : 7.

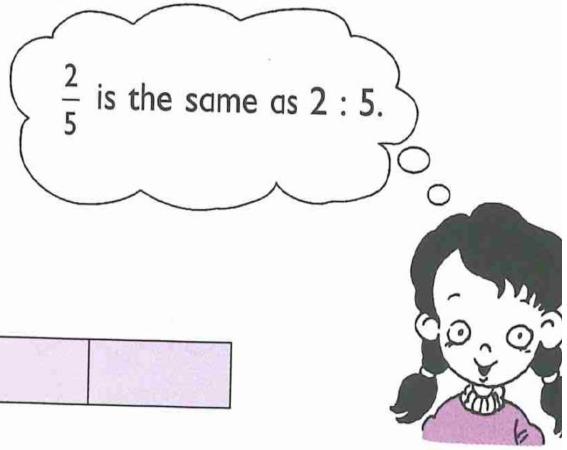


(c) The length of A is $\frac{2}{7}$ of the total length.

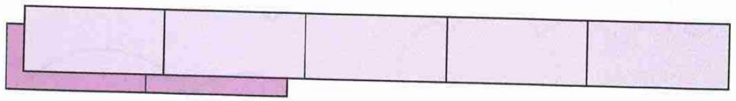
(d) The length of B is ▇ of the total length.

(e)

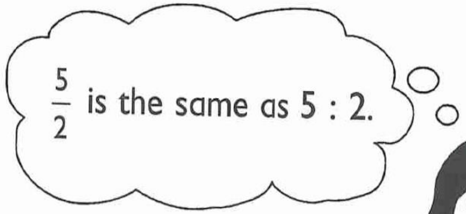
The length of A is $\frac{2}{5}$ of the length of B.



(f)



The length of B is $\frac{5}{2}$ of the length of A.

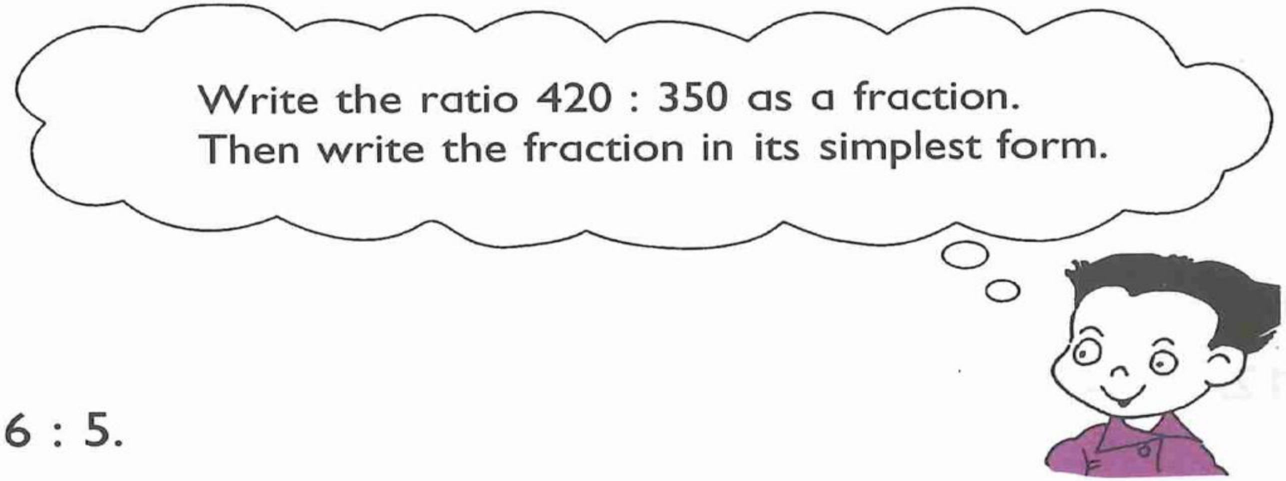


The length of B is ▇ times the length of A.

Meihua saved $420 and Sumin saved $350.

(a) Find the ratio of Meihua's savings to Sumin's savings.

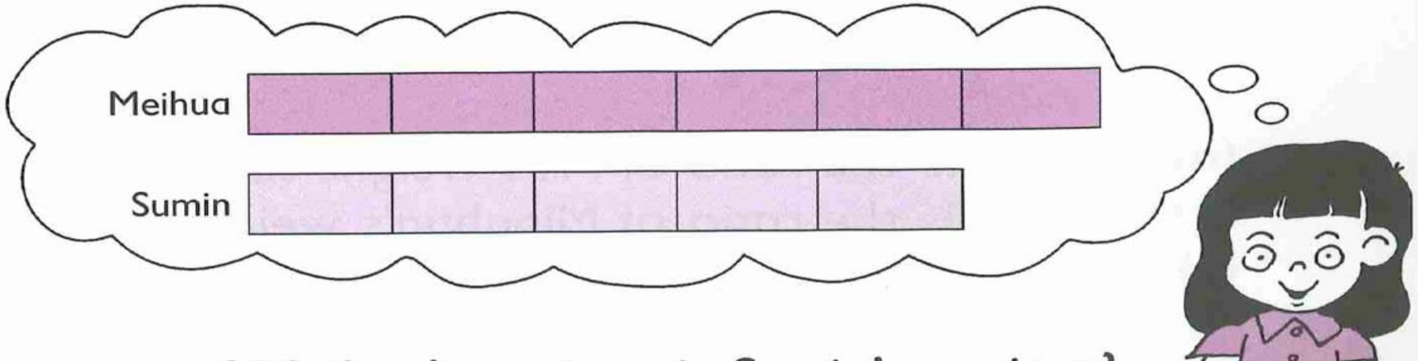
$$\frac{420}{350} = \frac{6}{5}$$



The ratio is 6 : 5.

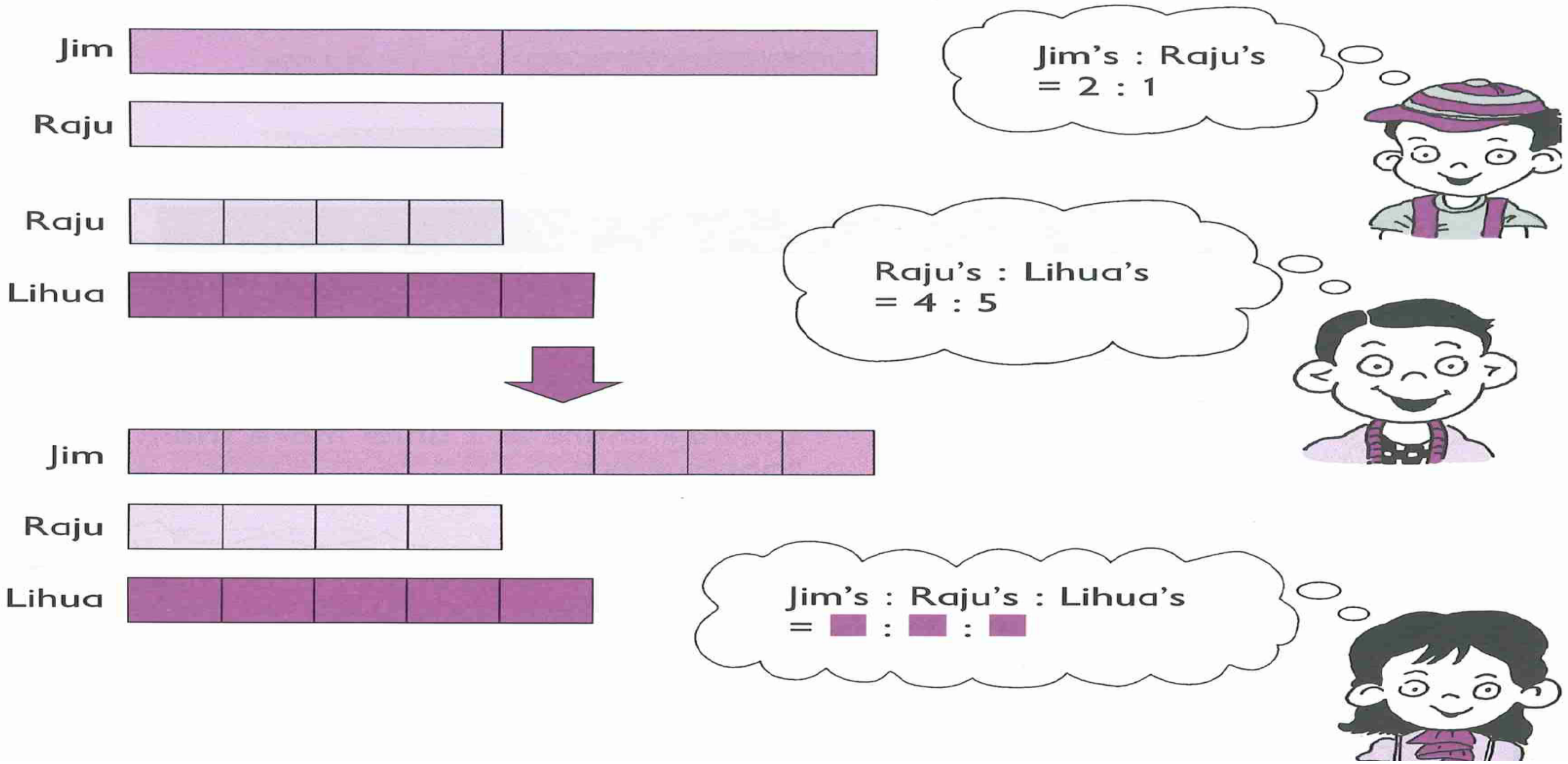
(b) What is the ratio of Sumin's savings to Meihua's savings?

(c) What fraction of Sumin's savings is Meihua's savings?

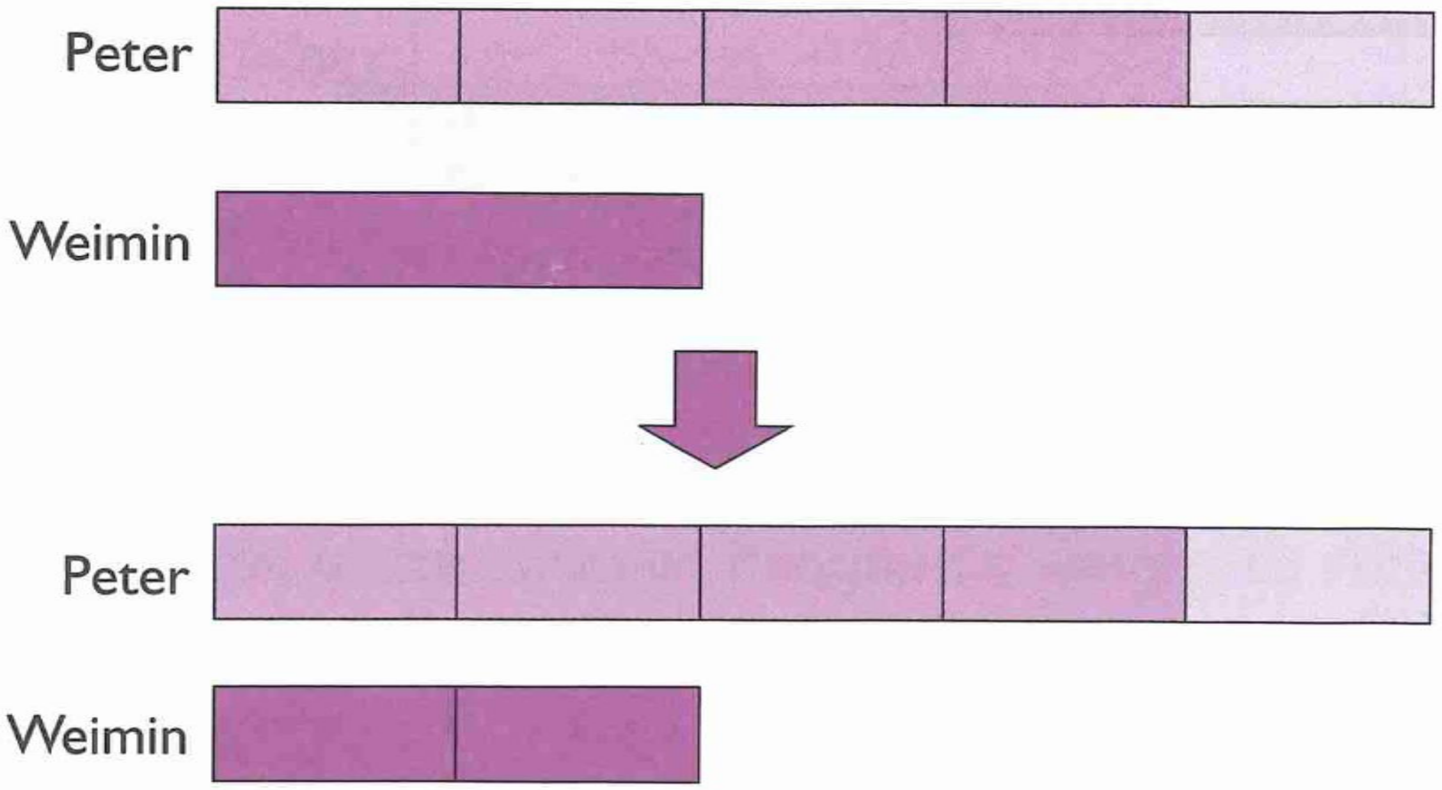


(d) What fraction of Meihua's savings is Sumin's savings?

The ratio of the number of Jim's marbles to Raju's is 2 : 1 and the ratio of the number of Raju's marbles to Lihua's is 4 : 5. Find the ratio of the number of Jim's marbles to Raju's to Lihua's.



$\frac{4}{5}$ of Peter's money is twice as much as Weimin's money. What fraction of Peter's money is Weimin's money?

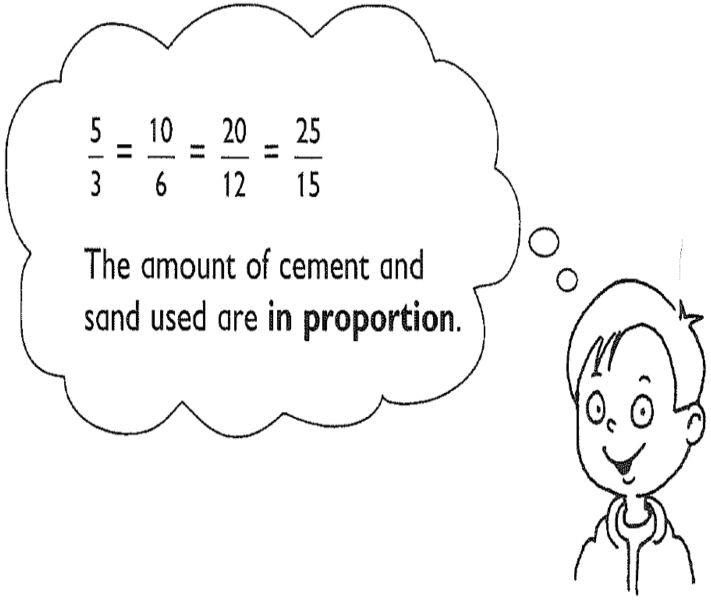


Workbook Exercise 11

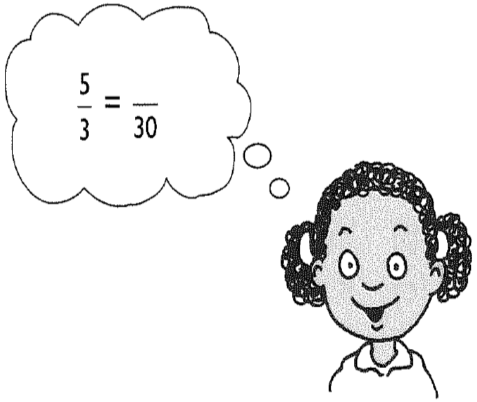
Ratio and Proportion

John used the following table to help him make a mixture of sand and cement.

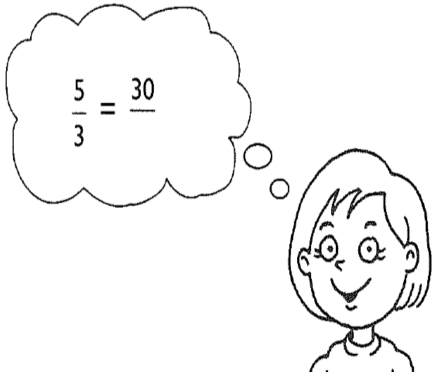
Number of buckets of cement	5	10	20	25
Number of buckets of sand	3	6	12	15



Using the same proportion, how many buckets of cement are needed to mix with 30 buckets of sand?



Using the same proportion, how many buckets of sand are needed to mix with 30 buckets of cement?



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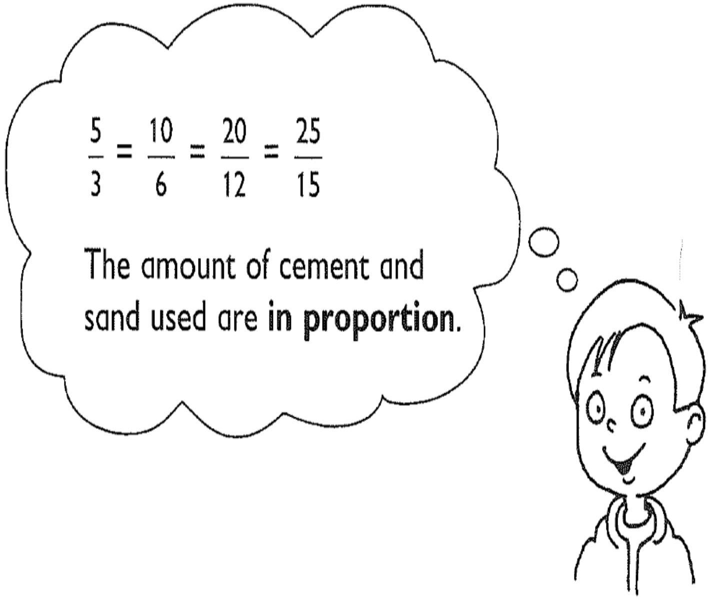
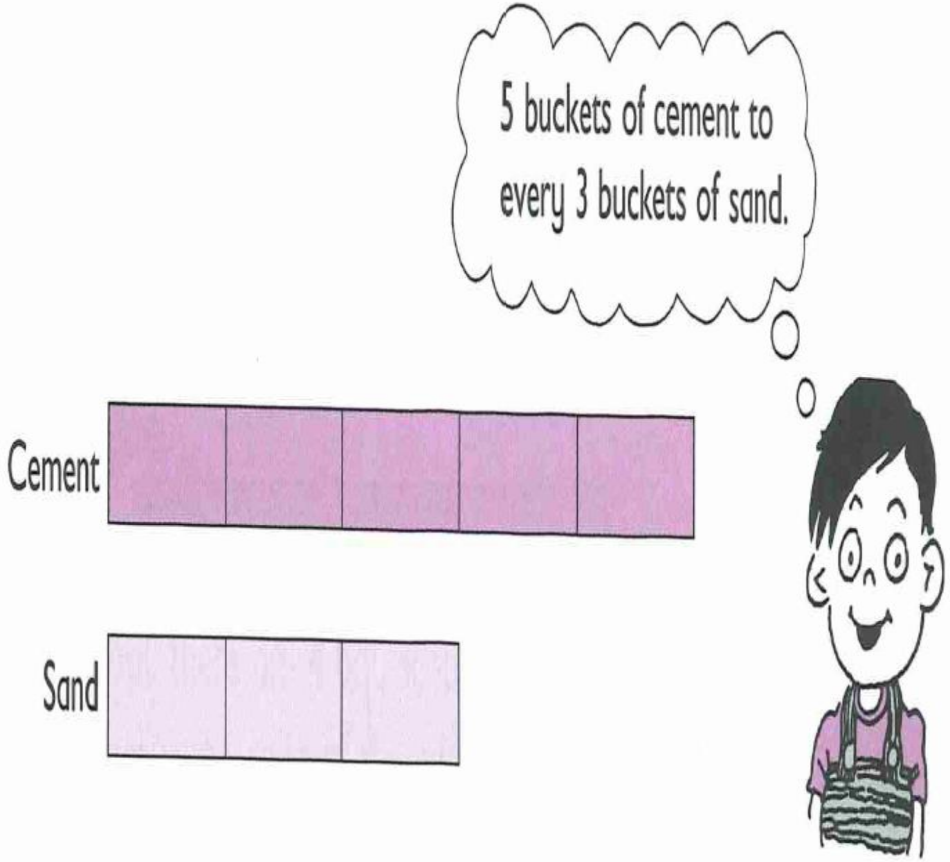
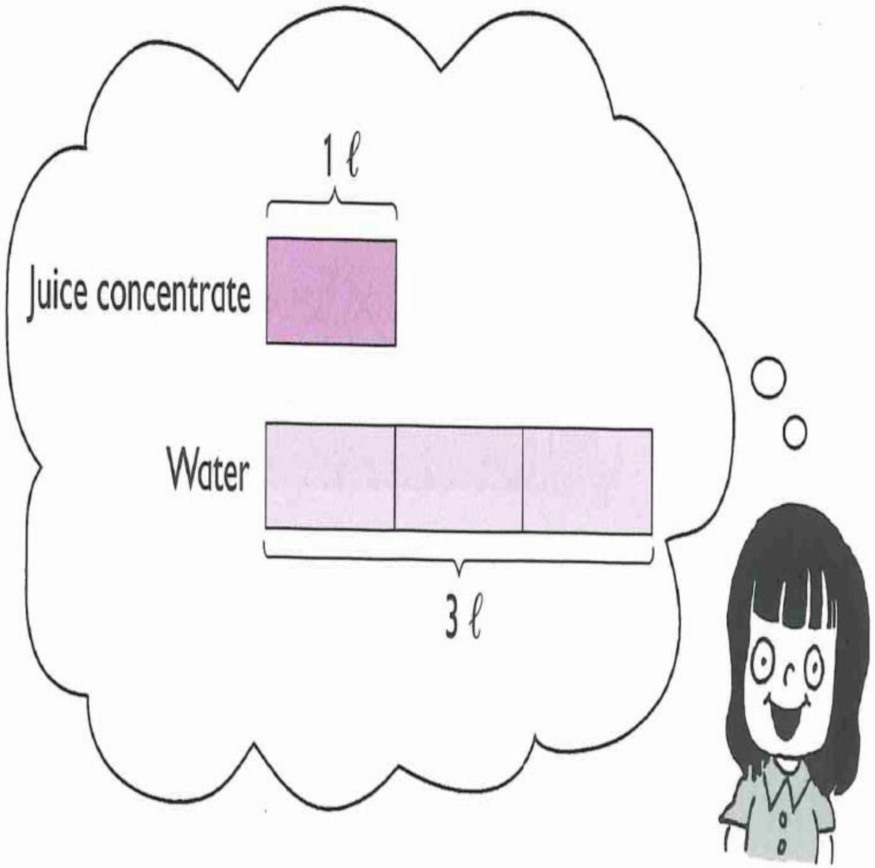


Cement and sand are mixed in the ratio 5 : 3.

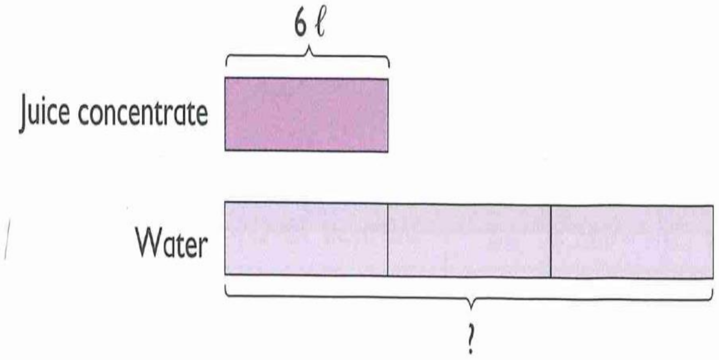


Wendy mixed 1 liter of juice concentrate with every 3 liters of water to make a drink for a party.

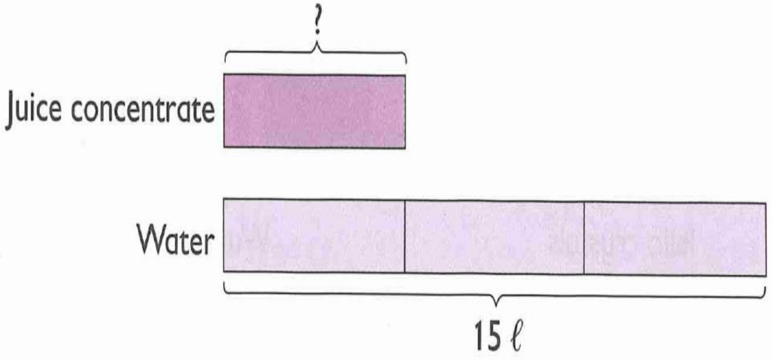
(a) Find the ratio of the amount of juice concentrate to the amount of water.



(b) To make the same drink, how many liters of water are needed to mix with 6 liters of juice concentrate?



(c) To make the same drink, how many liters of juice concentrate are needed to mix with 15 liters of water?



Changing Ratios

The ratio of the number of Peter's stamps to the number of Henry's stamps was 2 : 3. The ratio became 5 : 6 when Peter bought another 8 stamps.

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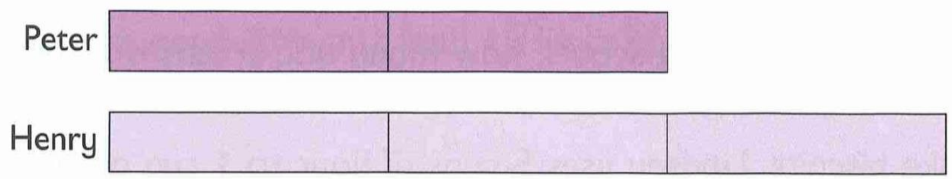
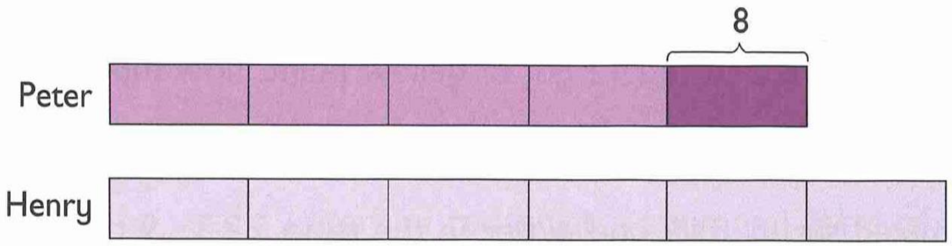
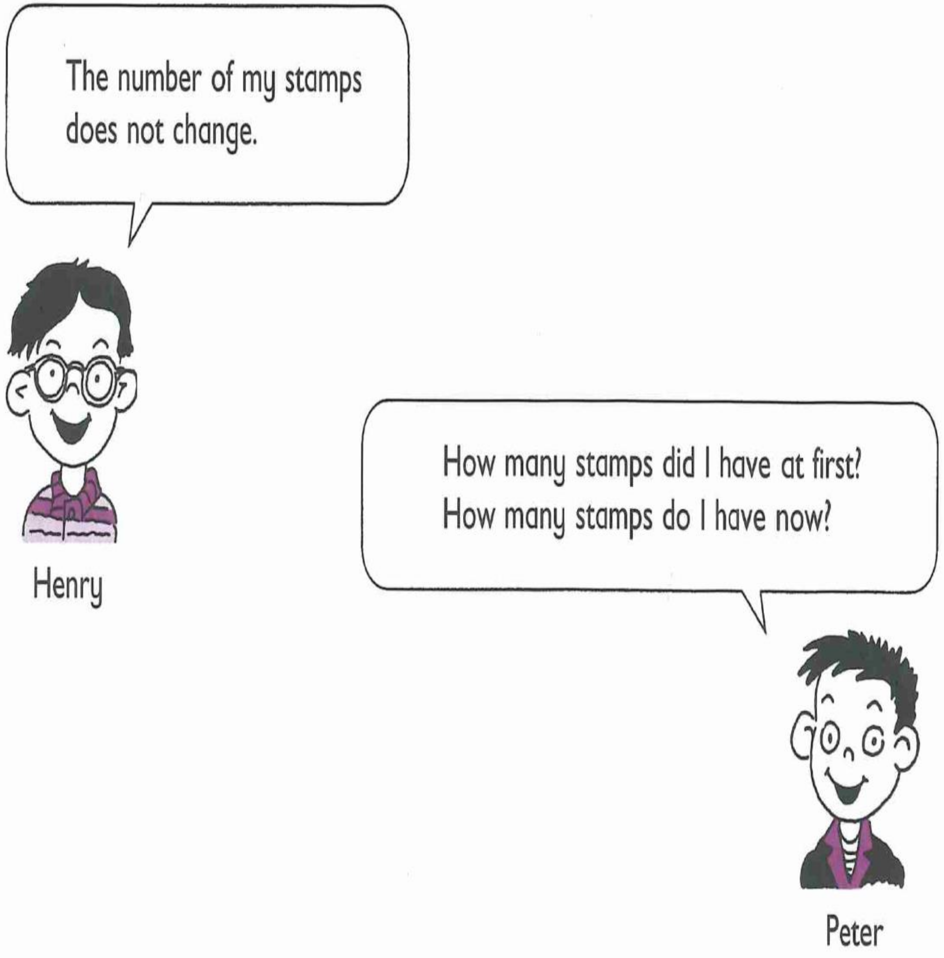


After:





The ratio of the number of Joe's stamps to Damon's is 2 : 7. Damon has 56 stamps. If Damon gives 8 stamps to Joe, what will be the new ratio of the number of Joe's stamps to Damon's?

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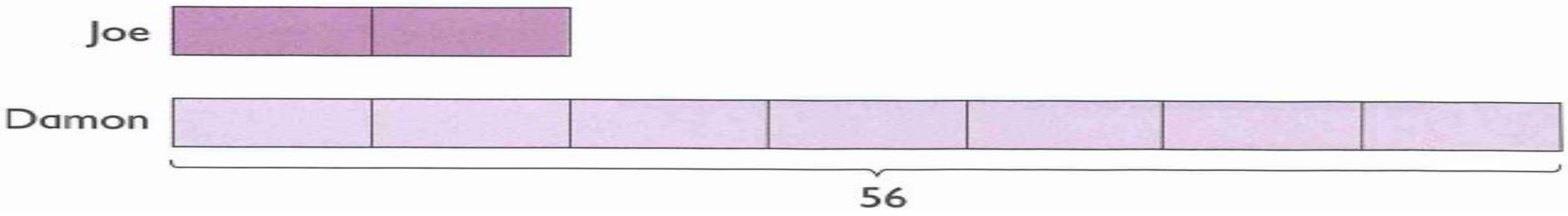


7 units = 56

1 unit = 56 ÷ 7 = 8

Number of Joe's stamps = 2 units = 8 × 2 = 16

After:

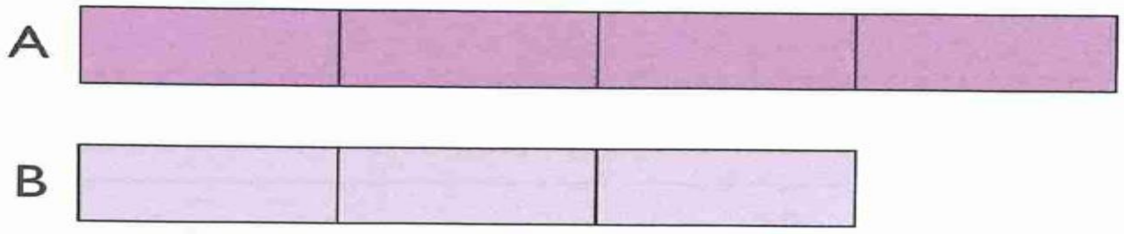
Number of Joe's stamps = 16 + 8 = 24

Number of Damon's stamps = 56 − 8 = 48

New ratio = 24 : 48 = ■ : ■

The ratio of the number of marbles in Box A to that in Box B is 4 : 3. If $\frac{1}{2}$ of the marbles in Box A are moved to Box B, what will be the new ratio of the number of marbles in Box A to that in Box B?

Before:



After:

The ratio of Susan's money to Mary's money was 5 : 3 at first. After Susan gave $20 to Mary, they had an equal amount of money each. How much money did Mary have at first?

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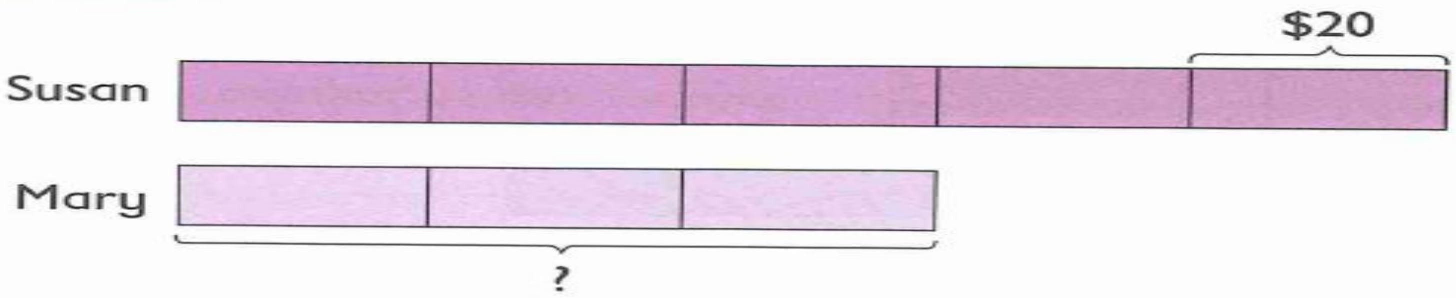

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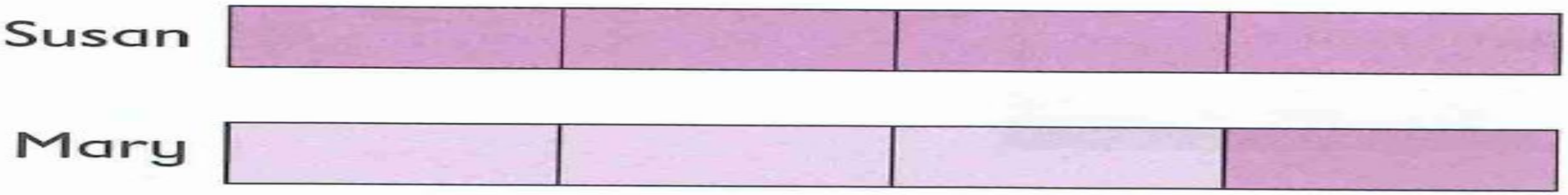

1 unit = $20

3 units = $▇

The ratio of Ian's money to Juan's money is 2 : 3. After spending $\frac{1}{2}$ of his money, Juan has $60 less than Ian. How much money does Ian have?

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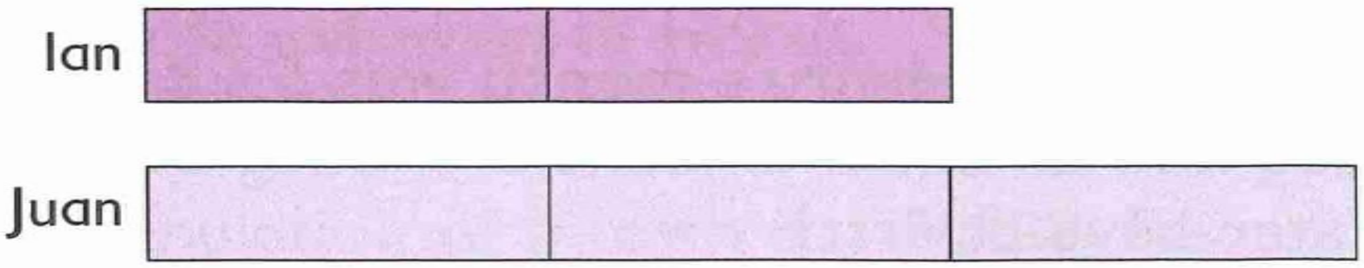

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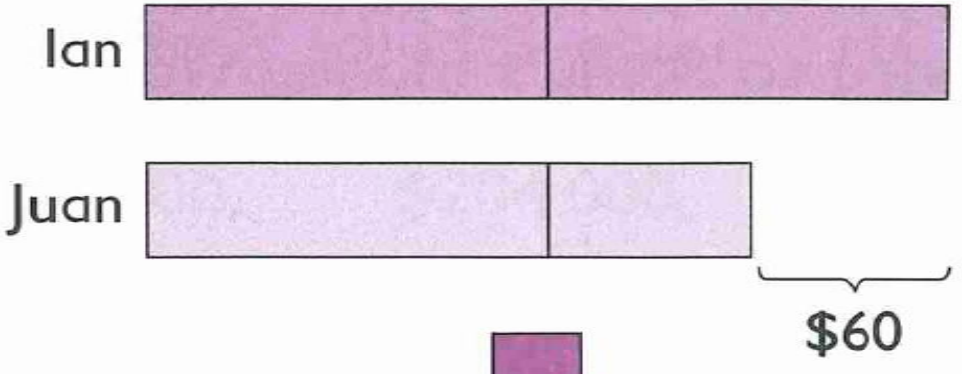


Percentages:

Part of a Whole as a Percentage

The table shows the enrollment in a music school.

Number of boys	240
Number of girls	360
Total number	600

What percentage of the students are boys?

Write 240 out of 600 as $\frac{240}{600}$ and then express it as a percentage.

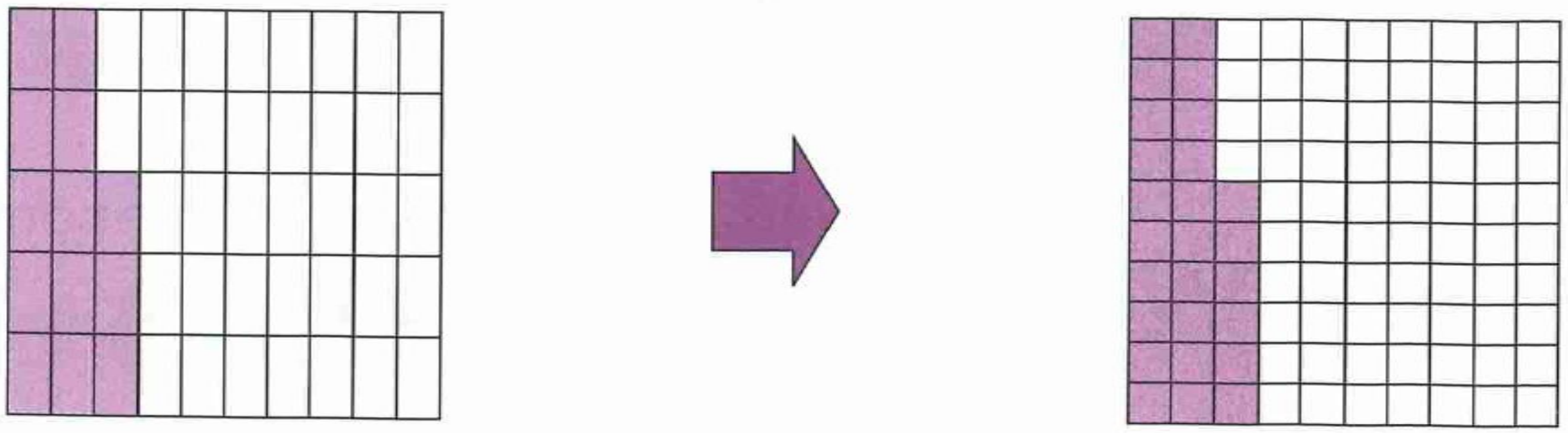
Method 1:

$$\frac{240}{600} = \frac{40}{100}$$
$$= 40\%$$

Method 2:

$$\frac{240}{600} \times 100\% = 40\%$$

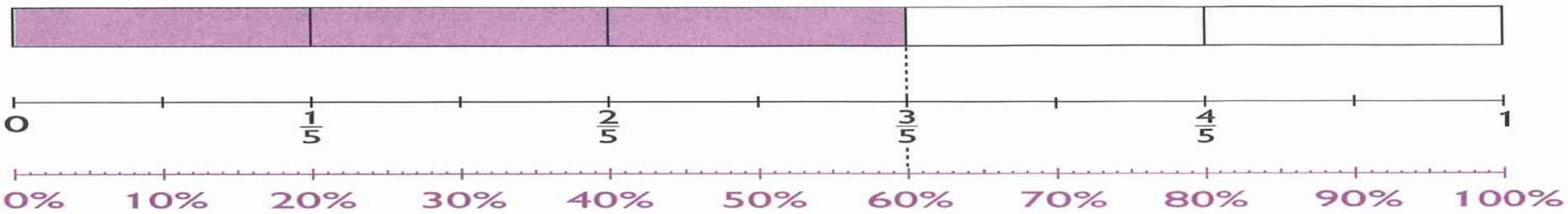
13 out of 50



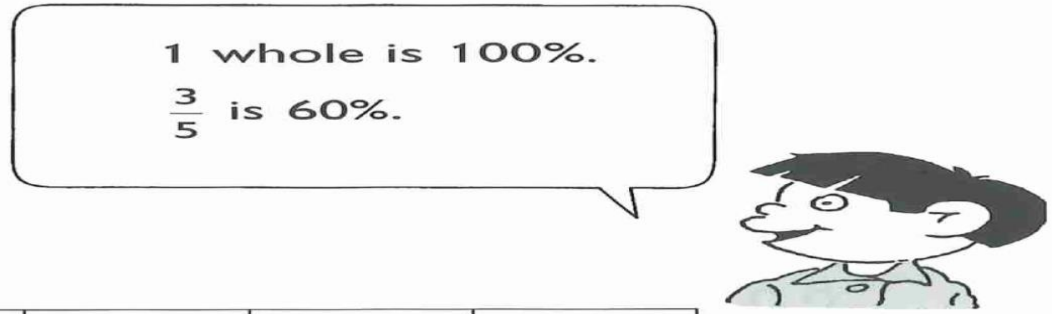
Write 13 out of 50 as $\frac{13}{50}$ and then express it as a percentage.

$$\frac{13}{50} = \blacksquare\%$$

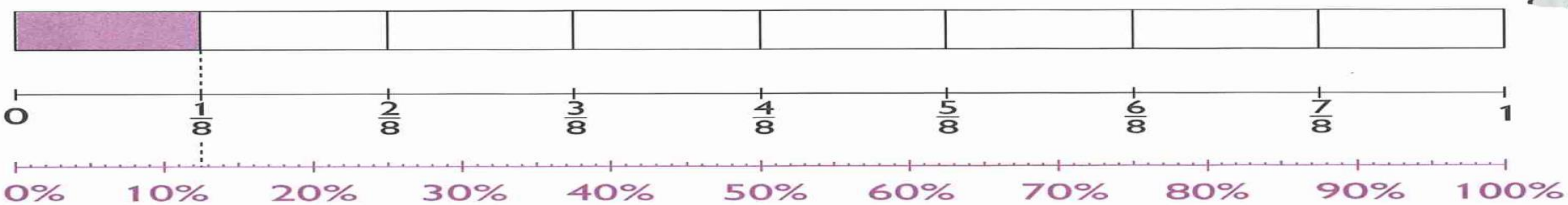
Express $\frac{3}{5}$ as a percentage.



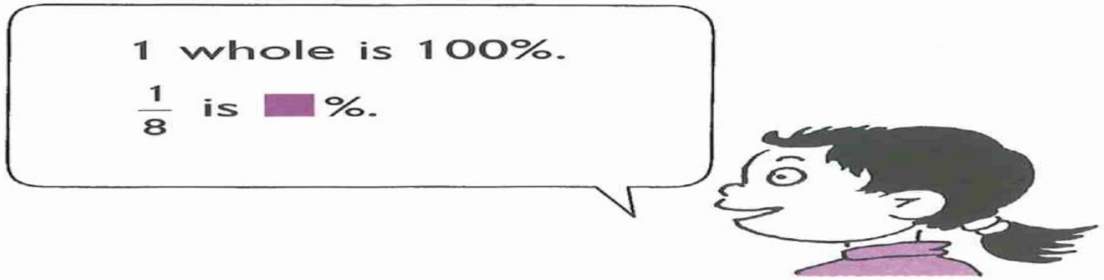
$\frac{3}{5} \times 100\% = 60\%$



Express $\frac{1}{8}$ as a percentage.



$\frac{1}{8} \times 100\% = \square\%$



Express 0.8 as a percentage.

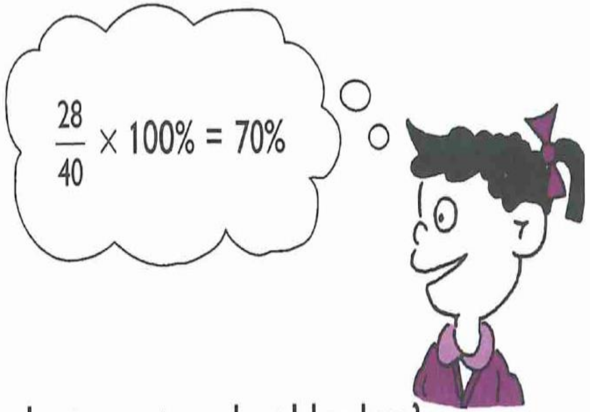
$0.8 \times 100\% = \square\%$

Express 0.075 as a percentage.

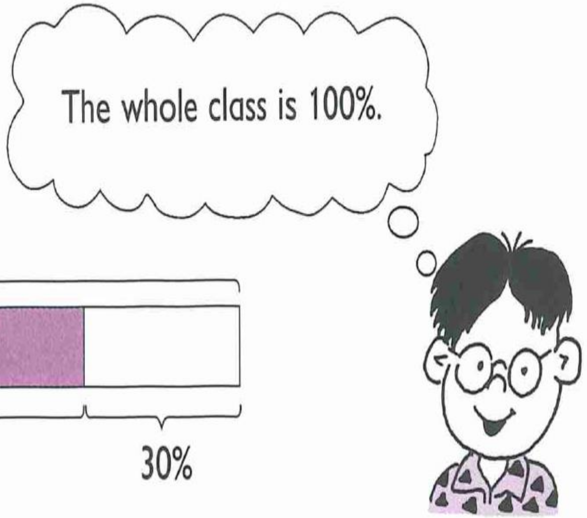
$0.075 \times 100\% = \square\%$

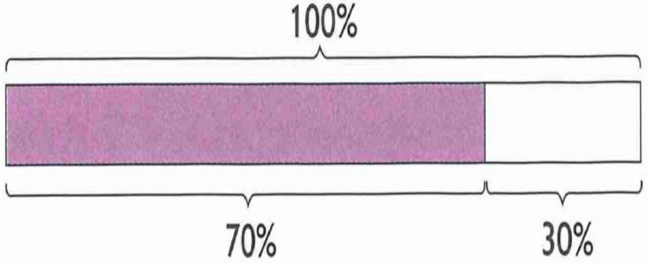
28 out of 40 students in a class walk to school. The rest go to school by bus.

(a) What percentage of the students walk to school?



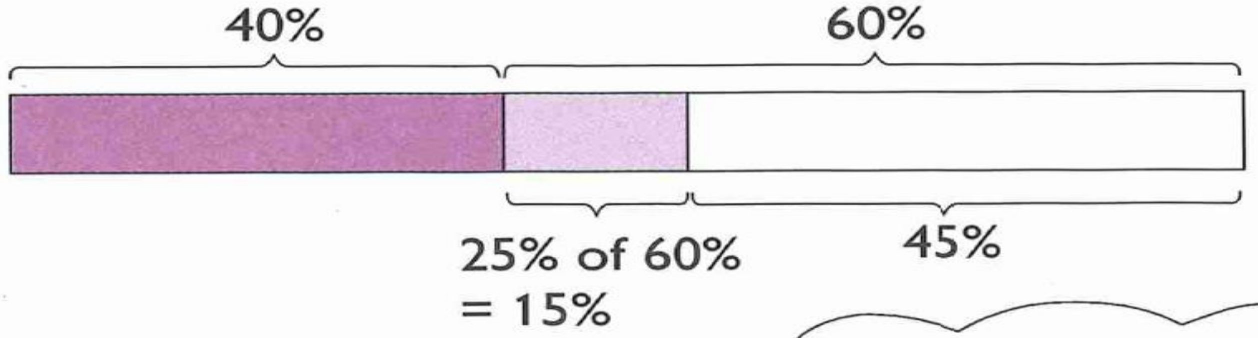
(b) What percentage of the students go to school by bus?

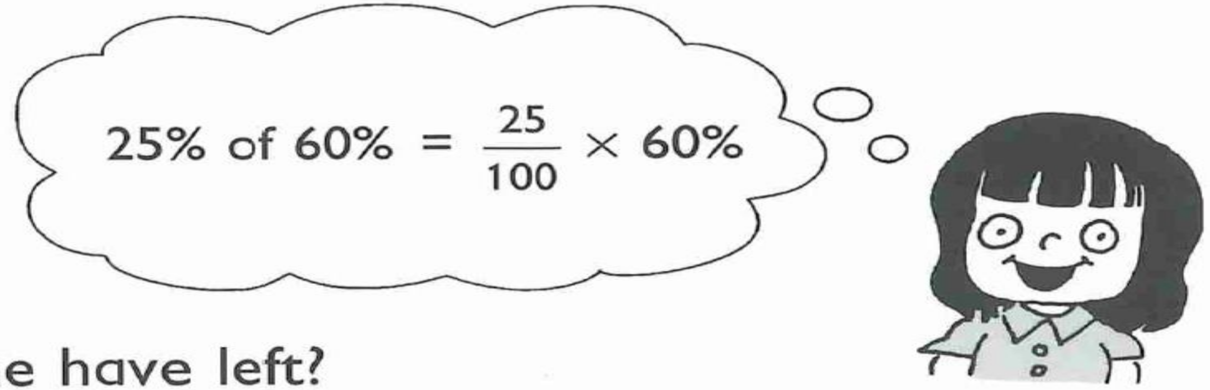




Ali had $120. He spent 40% of the money on a watch and 25% of the remainder on a pen.

(a) What percentage of his money did he spend?

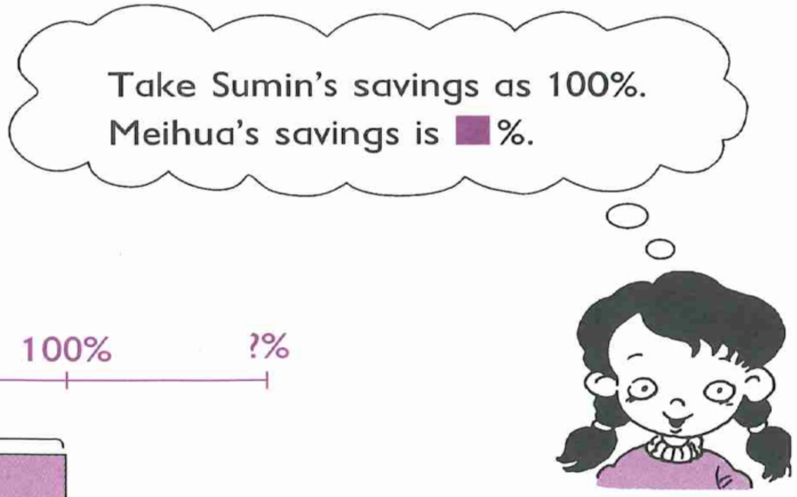


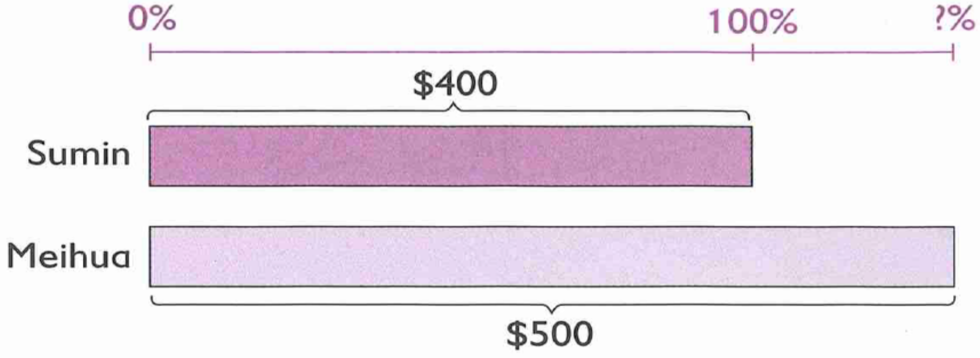


(b) How much money did he have left?

One quantity as the percentage of another

Sumin saves \$400 and Meihua saves \$500.
Express Meihua's savings as a percentage of Sumin's savings.





\$400 ⟶ 100%

\$400 ⟶ 100%

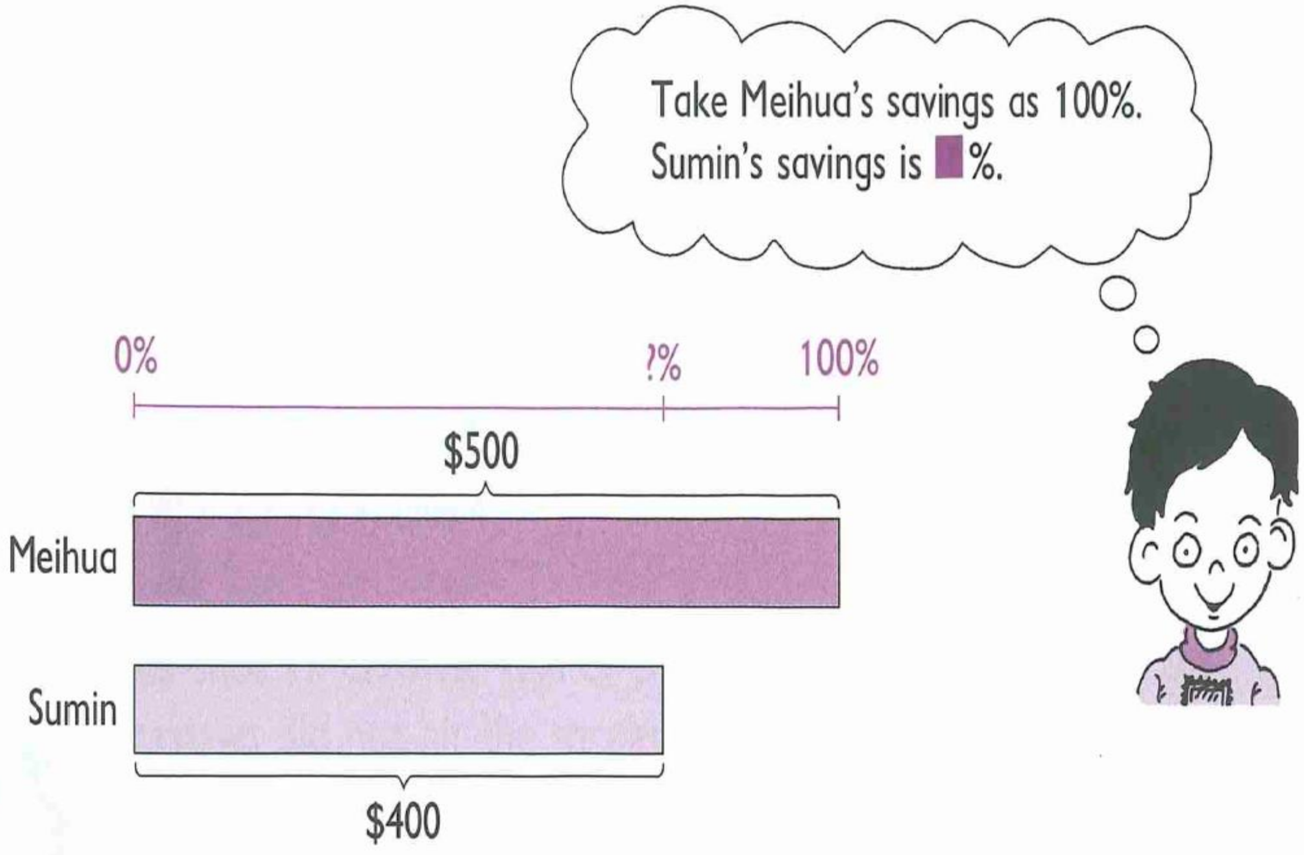
$$\$1 \longrightarrow \frac{100}{400}\%$$

$$\$500 \longrightarrow \frac{100}{400} \times 500\%$$

$$= 125\%$$

Meihua's savings is 125% of Sumin's savings.

Express Sumin's savings as a percentage of Meihua's savings.



$500 ⟶ 100%

$500 ⟶ 100%

$$\$1 \longrightarrow \frac{100}{500}\%$$

$$\$400 \longrightarrow \frac{100}{500} \times 400\%$$

$$= 80\%$$

Sumin's savings is 80% of Meihua's savings.

What percentage of \$3 is 30¢?

$$\frac{30}{300} \times 100\% = \blacksquare\ \%$$

30¢ is ■ % of \$3.

Express 1.35 m as a percentage of 90 cm.

$$\frac{135}{90} \times 100\% = \blacksquare\ \%$$

1.35 m is ■ % of 90 cm.

Express 300 ml as a percentage of 2 ℓ.

$$\frac{300}{2000} \times 100\% = \blacksquare\ \%$$

300 ml is ■ % of 2 ℓ.

The cost price of a television set is \$1200. It is sold for \$900. Express the selling price as a percentage of the cost price.

$$\frac{900}{1200} \times 100\% = \blacksquare\%$$

The selling price is ■% of the cost price.

The usual price of a vacuum cleaner is \$150. It is sold for \$120.

(a) How much is the discount?

$$\begin{aligned} \text{Discount} &= \$150 - \$120 \\ &= \$30 \end{aligned}$$

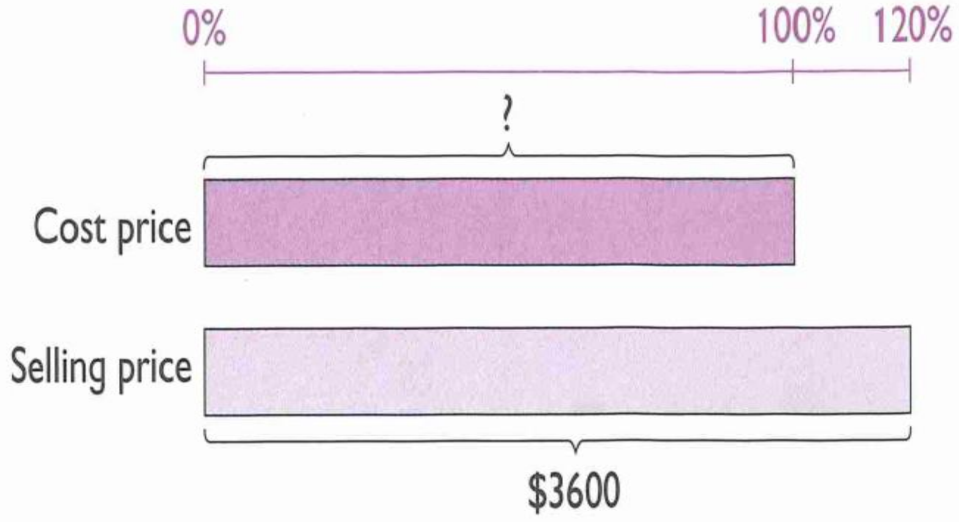
(b) Express the discount as a percentage of the usual price.

$$\frac{30}{150} \times 100\% = \blacksquare\%$$

The discount is ■% of the usual price.

Solve percentage problems by unitary method

Sean sells a set of furniture for $3600. The selling price is 20% more than the cost price. Find the cost price of the set of furniture.



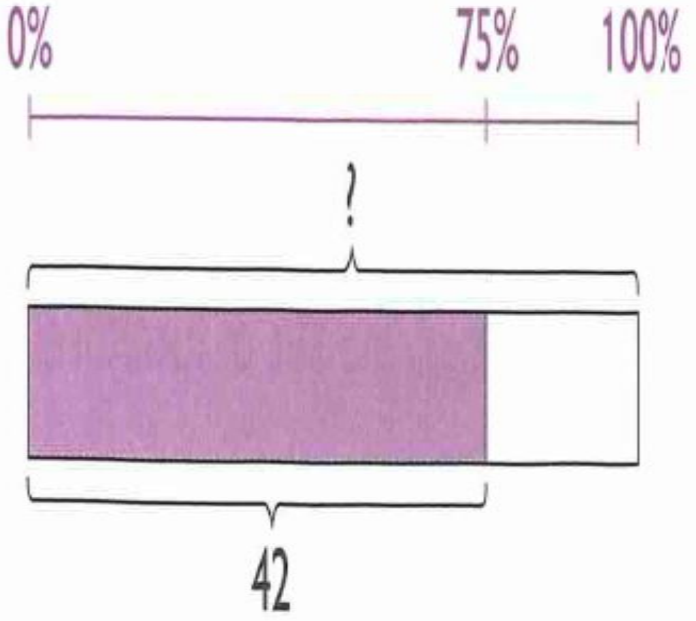
$$120\% \longrightarrow \$3600$$

$$1\% \longrightarrow \$\frac{3600}{120}$$

$$100\% \longrightarrow \$\frac{3600}{120} \times 100$$

$$= \$\blacksquare$$

Meili scored 42 points in a test. This was 75% of the total score. Find the total score.



75% ⟶ 42

75% ⟶ 42

$1\% \longrightarrow \frac{42}{75}$

$100\% \longrightarrow \frac{42}{75} \times 100 = \blacksquare$

The total score was ■.

Percent means per 100, or divided by 100. Dividing by 100 moves the decimal point two places to the left.

$$24\% = \frac{24}{100} = .24$$

•To convert a fraction or decimal to a percentage, multiply by 100:

Multiply the fraction by 100 to give the result as a percentage value.

$$\frac{1}{5} \times 100 = 5\overline{)100}\ \ 20$$

•To convert a percent to a fraction, divide by 100 and reduce the fraction (if possible):

Divide the percentage value by 100 and simplify the fraction if necessary.

$$60\% = \frac{60}{100} = \frac{3}{5}$$

•12 people out of a total of 25 were female. What percentage were female?

Multiply by 100. Dividing the top and bottom by 25 (cancelling) leaves 12 x 4.

$$\frac{12 \times \overset{4}{\cancel{100}}}{\cancel{25}} = 48\%$$

•The price of a $1.50 candy bar is increased by 20%. What was the new price?

Multiply the price by 20% (20/100). Add the result to the original price. ($1.50 + .30 = $1.80)

$$\$1.50 \times \frac{20}{100} = \$0.30$$

$$\$1.50 + \$0.30 = \$1.80$$

- The tax on an item is $6.00. The tax rate is 15%. What is the price without tax?

The price, p, times 15% (15/100) equals 6. Solve the equation by multiplying both sides by 100 and then dividing both sides by 15. The price without tax (P) is 40.

$$P \times \frac{15}{100} = 6$$

$$P \times \frac{15}{\cancel{100}} \times \cancel{100} = 6 \times 100$$

$$P \times \frac{\cancel{15}}{\cancel{15}} = \frac{600}{15} = 40$$

Decimals:

- Decimal numbers and fractions are two representations useful to express numbers and quantities that involve amount that is less than the unit of 1.
- We often need to express such quantities in the context of measurement, and that is the reason most Japanese elementary mathematics textbooks introduce decimal numbers and fractions in measurement contexts

Decimal	*Words*	*Fraction*
0.1	1 tenth	$\frac{1}{10}$
0.01	1 hundredth	$\frac{1}{100}$
0.001	1 thousandth	$\frac{1}{1000}$

(a) $0.7 = \frac{7}{10}$

(b) $0.09 = \frac{9}{100}$

(c) $0.004 = \frac{4}{1000} = \frac{1}{250}$

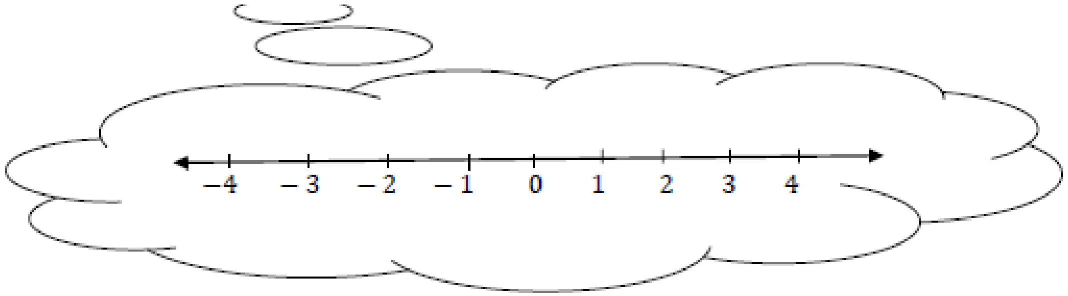
(d) $0.47 = \frac{47}{100}$

(e) $0.132 = \frac{132}{1000} = \frac{33}{250}$

(f) $1.75 = \frac{175}{100} = \frac{7}{4}$

What are rational numbers?

- Integers, I, is a set of numbers that include positive and negative numbers and zero.
- Imagine a number line
- These numbers are all integers. The set of integers does not include decimals or fractions.



Rational Numbers

Q, is any number that can be written in the form, $\frac{m}{n}$,

where m and n are both integers but $n \neq 0$.

Example: Using any two integers create a fraction and change to a decimal.

1a). $\frac{-6}{3} = \frac{-2}{1} = -2$ *** notice -2 is an integer and a rational number.

*** any integer can be written as a fraction using 1 as the denominator.

b). $\frac{2}{3} = 0.\overline{6}$ *** $0.\overline{6}$ is a repeating decimal and a rational number.

c). $\frac{7}{8} = 0.875$ *** 0.875 is a terminating decimal and a rational number.

d). $\frac{100}{25} = 4$ *** 4 is an integer and a rational number.

Therefore, rational numbers include all integers, fractions, terminating decimals and repeating decimals.

a). $\frac{-1}{4}$ $\longrightarrow$ Rational. It's a fraction. Even as a terminating decimal, -0.25 it's still rational.

b). $\sqrt{9}$ $\longrightarrow$ is 3. Rational. 3 is an integer.

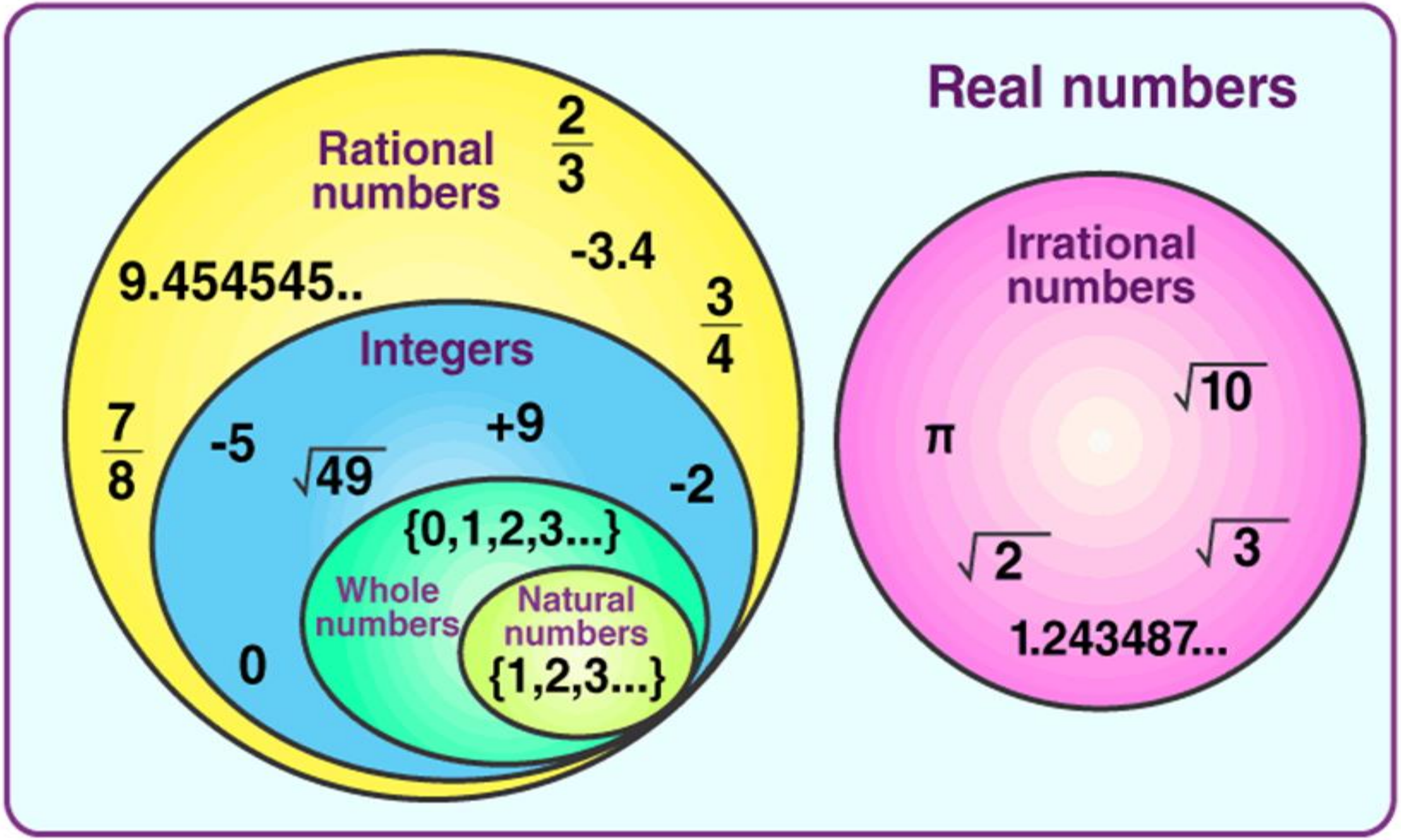
c). $\frac{-4}{-9}$ $\longrightarrow$ $\frac{4}{9}$ is rational, it's a fraction. Even as a repeating decimal, $0.\overline{4}$ it's still rational.

d). $\sqrt{75}$ $\longrightarrow$ $= 8.660254038...$

e). π $\longrightarrow$ $= 3.1415926535...$

These numbers are non-repeating and non-terminating decimals.

These types of numbers are called **irrational numbers**, $\overline{\mathbb{Q}}$.



Real numbers
Rational numbers
2/3
-3.4
9.454545..
3/4
7/8
Integers
-5
+9
√49
-2
{0,1,2,3...}
Whole numbers
Natural numbers
{1,2,3...}
0
Irrational numbers
√10
π
√2
√3
1.243487...

Properties of Rational and Irrational Numbers

- Here are some rules based on arithmetic operations such as addition and multiplication performed on the rational number and irrational number.

- **#Rule 1:** The sum of two rational numbers is also rational.
- Example: $1/2 + 1/3 = (3+2)/6 = 5/6$

- **#Rule 2:** The product of two rational number is rational.
- Example: $1/2 \times 1/3 = 1/6$

- **#Rule 3:** The sum of two irrational numbers is not always irrational.
- Example: $\sqrt{2}+\sqrt{2} = 2\sqrt{2}$ is irrational
- $2+2\sqrt{5}+(-2\sqrt{5}) = 2$ is rational

- **#Rule 4:** The product of two irrational numbers is not always irrational.
- Example: $\sqrt{2} \times \sqrt{3} = \sqrt{6}$ (Irrational)
- $\sqrt{2} \times \sqrt{2} = \sqrt{4} = 2$ (Rational)

To multiply decimals without a calculator, line-up the last decimal place. The number with the most digits should go on top. Don't worry about the sign until your final answer.

a). $(-1.5) \times 1.8 = ?$

This is negative

Workings:

$$\begin{array}{r} 1.5 \\ \underline{\times\ 1.8} \\ 120 \\ \underline{+\ 150} \\ 2.70 \end{array}$$

Move the decimal in two places in the final answer.

$$\begin{array}{r} 1.5 \\ \underline{\times\ 1.8} \end{array}$$

Answer: $-\ 2.70$

b). $(-\ 2.6\ ) \times (\ -\ 3.25) =$

This is positive

Workings:

$$\begin{array}{r} 3.25 \\ \underline{\times\ 2.6} \\ 1950 \\ \underline{+\ 6500} \\ 8.450 \end{array}$$

Move the decimal in three places in the final answer.

$$\begin{array}{r} 3.25 \\ \underline{\times\ 2.6} \end{array}$$

Answer : $+\ 8.450$

Dividing Decimals

a). $(-5.1) \div 3$ ⇒

$$\begin{array}{r} 1.7 \\ 3\,\overline{)\,5.1} \\ \underline{-3} \\ 2\,1 \\ \underline{-2\,1} \\ 0 \end{array}$$

Answer: -1.7

b). $\underline{(-7.5)}$
-5

⇒

$$\begin{array}{r} 1.5 \\ 5\,\overline{)\,7.5} \\ \underline{-5} \\ 2\,5 \\ \underline{-2\,5} \\ 0 \end{array}$$

Answer: 1.5

c). $(-10.5) \div 0.25$ ⇒ $0.25\,\overline{)\,10.5}$ ⇒ $25\,\overline{)\,1050.}$

$$\begin{array}{r} 42 \\ 25\,\overline{)\,1050.} \\ \underline{-100} \\ 5\,0 \\ \underline{-5\,0} \\ 0 \end{array}$$

You MUST move the decimal two places 0.25 becomes 25
Therefore the 10.5 must also be adjusted and become 1050.

Answer: -42 Don't forget to go back and look at the sign!

How to Convert a Decimal into a fraction

$$0.25 = \frac{0.25}{1} = \frac{0.25 \times 100}{1 \times 100} = \frac{25 \div 25}{100 \div 25} = \frac{1}{4}$$

Repeated decimal Fraction

- **Example: Convert 0.6666… Into fraction.**
- **Solution:** Let x = 0.6666
- Now multiply x by 10 on both sides.
- 10 x = 6.666…
- Subtracting x from 10x, we get;
- 10x-x = 6.666…-0.6666
- 9x = 6.000
- x = 6/9 = ⅔
- Hence, 0.6666… = ⅔

- If a car dealership gives a 5% discount on a car, the dealership will make a $5250 profit on the car.
- If, instead it will give a 25% discount, the dealership will lose $1750.

- How much did the dealership pay for the car (in dollars)?

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